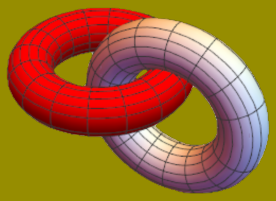
ABSTRACT BOOK
ATSF 2024 CONFERENCE

International Conference:

APPROXIMATION THEORY AND SPECIAL FUNCTIONS

ATSF 2024 - 8th SERIES



SEPTEMBER 4-7, 2024

TOBB UNIVERSITY OF ECONOMICS AND TECHNOLOGY

ANKARA - TÜRKİYE

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Preface

Dear Participants,

Welcome to the international conference entitled "Approximation Theory and Special Functions - ATSF 2024 Conference". This abstract book serves as your comprehensive guide to the abstracts of all papers presented at the conference. This book contains a total of 172 abstracts, including 165 oral presentations and 7 poster presentations.

Hosted by TOBB University of Economics and Technology (Ankara, Türkiye), the ATSF 2024 Conference has taken place from September 4th to 7th, 2024. This esteemed institution has provided an ideal venue for our gathering, offering an enriching environment for learning, networking, and collaborative exploration.

The primary focus of this conference is on approximation theory, special functions, and their diverse applications. Mathematics in these areas continues to evolve with significant breakthroughs and practical implications. Through insightful presentations and discussions, we aim to delve into these advancements and foster new insights. We are pleased to announce that over 200 scientists from more than 30 different countries around the world have participated in the conference. This diverse participation underscores the global interest and importance of the topics we are addressing.

We extend our heartfelt gratitude to our invited speakers, special session organizers, members of the international advisory and scientific committee, and everyone involved in the organization. We would also like to thank TOBB ETU (TOBB University of Economics and Technology), TUBITAK (The Scientific and Technological Research Council of Türkiye), and TUBA (Turkish Academy of Sciences) for their generous support.

Thank you for joining us in this intellectual journey of exploration and knowledge sharing.

Best regards,

Prof. Dr. Oktay Duman
TOBB University of Economics and Technology
Department of Mathematics
Ankara, Türkiye
(Conference Co-Chair)

Prof. Dr. Esra Erkus-Duman
Gazi University
Department of Mathematics
Ankara, Türkiye
(Conference Co-Chair)

The ATSF 2024 Conference is partially supported by:

Invited Talks

Contributed Talks (*sorted by presenting author name*)

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Poster Presentations (*sorted by presenting author name*)

(Invited Talk)

Trigonometric Background Multivariate Smooth Poisson-Cauchy Singular Integrals Approximation

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Abstract

In this work we apply the uniform and L_p, $1 \leq p < \infty$, approximation properties of general smooth multivariate singular integral operators over $\mathbb{R}^N$, $N \geq 1$, studied in [1, 2, 3]. It is a trigonometric based approach with detailed applications to the corresponding smooth multivariate Poisson-Cauchy singular integral operators. The results are quantitative via Jackson type inequalities involving the first uniform and L_p moduli of continuity.

Keywords: Multivariate singular integral operator, multivariate modulus of continuity, multivariate Poisson-Cauchy operator, uniform and L_p approximation.

References:

[1] G. A. Anastassiou, Intelligent Mathematics: Computational Analysis, Springer, Heidelberg, New York, 2011.

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(Invited Talk)

Approximation by Parametrized Logistic Activated Convolution Type Operators

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Abstract

In this work we introduce for the first time the univariate parametrized logistic activated convolution type operators in three kinds. We present their approximation properties, that is the quantitative convergence to the unit operator via the modulus of continuity. We continue with the global smoothness preservation of these operators. We present extensively the related iterated approximation, as well as, the simultaneous approximation and their combinations. Including differentiability and fractional differentiabily into our research, are producing higher speeds of approximation. Simultaneous global smoothness preservation is also treated.

Keywords: Richards' curve function, parametrized logistic function, convolution type operator, Caputo fractional derivative, quantitative approximation, global smoothness preservation, simultaneous approximation, iterated approximation.

References:

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(Invited Talk)

Multinode Shepard Method: From Surface Reconstruction to Numerical Solution of PDEs

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Abstract

In 1985, Little announced the possibility of improving the approximation qualities of the classic Shepard interpolant using a convex combination of Shepard-like basis functions based on triangles with local linear polynomial interpolants based on the vertices of those triangles. Similarly to the Shepard basis functions, the triangular Shepard basis functions are the normalization of the product of the inverse distances from the vertices of the triangles. Starting from the 2016 paper [1], a new light is given to the triangular Shepard method by proving its quadratic approximation order theoretically and computationally and enlarging its applicability to "compact triangulations". After this insight, the ideas in Little's paper are realized, and they lead to the introduction of the multinode Shepard method [2], a powerful method for interpolation of scattered data based on inverse distance weighting and local polynomial interpolants on sets of unisolvent nearby nodes. The multinode Shepard method applies to the problem of surface reconstruction and the numerical solution of PDE via collocation [3]. After the appropriate modifications, the multinode Shepard method is suitable for interpolating scattered data on the sphere.

Keywords: Multinode Shepard method, scattered data approximation, surface reconstruction, numerical solution of PDEs, interpolation on the sphere.

References:

[1] F. Dell'Accio, F. Di Tommaso and K. Hormann, On the approximation of the triangular Shepard interpolation, IMA J. Numer. Anal. 36 (2016), 359–379.

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(Invited Talk)

Best Approximation by Polynomials on Regular Domains

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Abstract

This talk aims to explain a framework developed in [1] for characterizing weighted best approximation by polynomials on a family of regular domains, which includes spheres, balls, simplexes, and conic domains in $\mathbb{R}^d$. Each domain Ω in the family is equipped with a weight function W and the orthogonal polynomials in $L^2(\Omega, W)$ are assumed to possess:

- A differential operator with orthogonal polynomials as eigenfunctions, called spectral operator.
- A closed-form formula for the reproducing kernels of orthogonal polynomials of degree n, called addition formula.

With the help of the highly localized kernels derived via the addition formula [1, 2], the framework provides a unified approach for a characterization of the best polynomial approximation in the weighted L^p norm in terms of a K-functional defined via the spectral operator, or in terms of an equivalent modulus of smoothness.

Keywords: Multivariate singular integral operator, multivariate modulus of continuity, multivariate Poisson-Cauchy operator, uniform and L_p approximation.

References:

[1] Y. Xu, Approximation and localized polynomial frame on conic domains, J. Funct. Anal. 281 (2021), no. 12, Paper No.109257, 94 pp.

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(Invited Talk)

Multivariate Aldaz-Kounchev-Render Operators and Their Approximation Properties

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Abstract

The approximation properties of the Aldaz-Kounchev-Render (AKR) operators have investigated in several papers. We improve some existing quantitative results concerning these approximation properties. In particular, we describe classes of functions for which these operators approximate better than the classical Bernstein operators and classes of functions for which Bernstein operators approximate better than AKR operators. The new results, in particular involving monotonic convergence and Voronovskaja type formulas, are then extended to the bivariate case on the square $[0,1]^2$ and compared with other existing results.

We introduce the Aldaz-Kounchev-Render operators on a multidimensional simplex. In the case of the unit simplex of $\mathbb{R}^m$ these operators preserve the functions $1, x_1^j, \ldots, x_m^j$, where j is a positive integer. The Voronovskaja formula, the behaviour with respect to the convex function, and the limit of iterates of the operators are investigated.

Keywords: Aldaz-Kounchev-Render operators, Bernstein operators, iterates of Markov operators, convex functions.

References:

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(Invited Talk)

Interpolation Projections in Function Fréchet Algebras

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Abstract

We consider short exact sequences of function Fréchet algebras. If the factor space is defined by a sequence $(z_k)_{k=1}^{\infty}$, then the inverse of the epimorphism can naturally be represented as an interpolation operator.

Interestingly, the geometric conditions for the continuity of the corresponding interpolation projections in the spaces of analytic functions and in the spaces of infinitely differentiable functions are opposite in the following sense. For the spaces of analytic functions, arbitrary rapid convergence of $(z_k)_{k=1}^{\infty}$ to a boundary point is allowed, whereas there is an upper limit on the rate of convergence for such sequences in the second case.

Keywords: Interpolating projections, splitting of short exact sequences, spaces of analytic and infinitely differentiable functions.

(Invited Talk)

Approximating Fractional Calculus Operators with General Analytic Kernel by Stancu Variant of Modified Bernstein-Kantorovich Operators

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Abstract

The main aim of this paper is to approximate the fractional calculus (FC) operator with general analytic kernel by using auxiliary newly defined linear positive operators. For this purpose, we introduce the Stancu variant of modified Bernstein-Kantorovich operators and investigate their simultaneous approxi mation properties. Then we construct new operators by means of these auxiliary operators, and based on the obtained results, we prove the main theorems on the approximation of the general FC operators. We also obtain some quanti tative estimates for this approximation in terms of modulus of continuity and Lipschitz class functions. Additionally, we exhibit our approximation results for the well-known FC operators such as Riemann-Liouville integral, Caputo derivative, Prabhakar integral, and Caputo-Prabhakar derivative.

Keywords: Caputo derivative, modified Bernstein–Kantorovich operators, modulus of continuity, Prabhakar operator, Riemann–Liouville operator.

References:

[1] M. A. Özarslan and C. Kurt, Bivariate Mittag-Leffler functions arising in the solutions of convolution integral equation with 2D-Laguerre–Konhauser polynomials in the kernel, Appl. Math. Comput. 347 (2019), 631–644.

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On the Quasi-Orthogonality and Hahn-Classical Character

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Abstract

We shall focus on the quasi-orthogonality between two d-orthogonal polynomials sequences and we give some of their characterizations in terms of linear combinations and the left product of vector of linear forms by matrix polynomial. Therefore, structure relations for Hahn-classical d-orthogonal polynomials are provided.

Keywords: Orthogonal polynomials, quasi-orthogonality, Hahn-classical character, structure relation.

References:

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$(X, Y; \Phi, r, s)$-Admissible Quasilinear Operators on Generalized Morrey-Banach Spaces

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Abstract

The aim of this talk is to give the definitions of $(X, Y; \Phi, r, s)$-admissible quasilinear operators and the generalized Morrey-Banach spaces $M_{Y,\psi}$. We define these spaces for two Banach subspaces X, Y, a positive and measurable function Φ and non-increasing function ψ defined on $\mathbb{R} \times (0, \infty)$, and a wide range of the numerical parameters r and s, $1 \leq r < \infty$, $0 < s \leq \infty$. And also, we define the operator $\mathcal{S}_{\Phi,r,s}$ and we show that it is the generalization of all classical operators of harmonic analysis.

Furthermore, we prove the boundedness of $(X, Y; \Phi, r, s)$-admissible quasilinear operators $\mathcal{T}$ from the one generalized Morrey-Banach space M_{X,ψ_1} to another one M_{Y,ψ_2}, and also including weak estimates. The definition of $(X, Y; \Phi, r, s)$-admissible quasilinear operator $\mathcal{T}$ covers all previously given definitions of admissible operators. These conditions satisfy almost all the classical operators in harmonic analysis. Hence, we show that the maximal operator M, singular integral operator T, fractional maximal operator M_α and Riesz potential operator I_α as some applications of $(X, Y; \Phi, r, s)$-admissible quasilinear operators. The all theorems in the references [1, 2, 3, 4, 5, 6] are given as corollaries of $(X, Y; \Phi, r, s)$-admissible quasilinear operators.

Keywords: Admissible operators, classical operators of harmonic analysis, generalized Morrey-Banach spaces.

References:

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A Study of Fractional Differential Equation with Integral Boundary Conditions

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Abstract

In this paper, we study the existence and uniqueness of a solution for a fractional order fractional differential equations with integral conditions. Using the method of energy inequalities, we need a priori estimate and the density of the range of the operator generated by the given problem.

Keywords: Time fractional differential equation, integral conditions, strong solution.

References:

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Normal Subgroups of the Unitary and General Linear Groups of Certain C^*-Algebras

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Abstract

In this paper, we extend the results of the second author for involutions, projections and unitary groups in [1] to the case of symmetries, idempotents and general linear groups. The type of a symmetry s is the element $[p]$ in $K_0(\mathcal{A})$, where $s = 1 - 2e$, $e \in \mathcal{I}(\mathcal{A})$, and $e \sim_s p$, for $p \in \mathcal{P}(\mathcal{A})$. As, $K_0(\mathcal{O}_n) \simeq \mathbb{Z}_{n-1}$, the type of a symmetry is associated with an integer. A symmetry s is called of even type, if $[p] = 2k$ and odd type, if $[p] = 2k + 1$. We show that a normal subgroup $\mathcal{N}$ of $GL(\mathcal{O}_n)$, $n < \infty$ contains all the symmetries, if,

1. $\mathcal{N}$ contains a symmetry of the type 1 (i.e. of type [1]), or
2. $\mathcal{N}$ contains a non-trivial symmetry, and $(n - 1)$ is a prime number, or
3. $\mathcal{N}$ contains a non-trivial symmetry such that its type and $(n - 1)$ are relatively prime.

Then, using M. Leen's result [2], we deduce that $GL_0(\mathcal{O}_n) \subseteq \mathcal{N}$. In the case of $\mathcal{O}_\infty$, we prove that if $\mathcal{N}$ contains a symmetry of odd type, then $\mathcal{N}$ contains all the symmetries of $\mathcal{O}_\infty$. Consequently, by Leen's result, we get that $GL_0(\mathcal{O}_\infty) \subseteq \mathcal{N}$.

Also, we show that if $\mathcal{N}$ is a normal subgroup of unitary groups of compact operator $\mathbb{K}$, which contains some certain type of involution, then $\mathcal{N}$ contains all the involutions of $\mathbb{K}$. Further, we prove that if $\mathcal{N}$ is a normal subgroup of general linear groups of compact operator $\mathbb{K}$, which contains some certain type of symmetry, then $\mathcal{N}$ contains all the symmetries of $\mathbb{K}$.

Keywords: Involutions, symmetries, unital C^*-algebras.

References:

[1] A. Al-Rawashdeh, Normal generation of unitary groups of Cuntz algebras by involutions, Acta Math. Univ. Comenian. (N.S.) 77 (2008), no. 1, 1–7.

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Hypergeometric Function on α-Time Scales

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Abstract

This talk is devoted to introduce the beta function and hypergeometric function on α-time scale. For this purpose, we first present the α-time scale [1, 2] based on α-operator which is a convex combination of delta and nabla shift operators of (q, h)-time scales. The α-hypergeometric function comprises many other special functions as specific or limiting cases and it unifies and extends continuous and discrete analogues of hypergeometric functions such as (q, h)-hypergeometric and q-hypergeometric functions. Moreover, we demonstrate the relations with α-gamma and α-beta functions.

Keywords: α-time scale, beta function, gamma function, hypergoemetric function, binomial series.

References:

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Bilinear Multipliers on Orlicz Spaces on Locally Compact Groups

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Abstract

In this work, we generalize bilinear multipliers acting on Orlicz spaces on $\mathbb{R}^n$ to Orlicz spaces acting on locally compact abelian groups G. We focus on describing these bilinear multipliers from the point of view of abstract harmonic analysis. We obtain separate necessary and sufficient conditions for the existence and boundedness of such bilinear multipliers.

Keywords: Bilinear multipliers, Orlicz spaces, locally compact Abelian groups.

References:

[1] A. Osançlıol and S. Öztop, Weighted Orlicz algebras on locally compact groups, J. Aust. Math. Soc. 99 (2015), no. 3, 399–414.

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Approximation by Dilations in Spaces of Analytic Functions

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Abstract

We study some weighted Banach spaces of holomorphic functions of several complex variables both in the polydisk and in the unit ball. We aim to find Mergelyan-type conditions on the weight function to guarantee that the dilations of a given function converge to the same function in norm. In particular, we obtain conditions on the weights to ensure that the analytic polynomials are dense in the given space of holomorphic functions.

Keywords: Bergman space, Besov space, dilation, weight function.

References:

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An r-Modified Crank-Nicolson Difference Scheme for the Numerical Solution of a Source Identification Problem

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Abstract

In this study, we consider a time-dependent source identification problem governed by a one-dimensional, linear parabolic equation. An r-modified Crank-Nicolson difference scheme, which is a combination of the well-known finite difference schemes Rothe and the standard Crank-Nicolson difference schemes, is proposed for the numerical solution of the problem [1, 2]. Moreover, some stability estimates for the solution of the source identification problem and the solution of the proposed difference scheme are established by using some tools from the operator theory [3]. A detailed numerical analysis is given by employing the proposed difference scheme on a model problem.

Keywords: Inverse problem, numerical solution, stability.

References:

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Korovkin-Type Theorems via Certain Semi-Convergences

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Abstract

This work focuses on the Korovkin-type theorems that are dependent on the semi-types of 'exhaustiveness' and 'almost uniform convergence'. Since it is known that the convergence types mentioned above are between pointwise and uniform convergence, it will be noticed that the circumstances can be mitigated in the classical Korovkin's Theorem.

Keywords: Function sequences, convergence, Korovkin theorem.

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Pairs of Closed Geodesics in Metric Graphs

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Abstract

In this paper we study counting pairs of closed geodesics on metric graphs. We obtain an asymptotic for the number of pairs of closed geodesics ordered by their word lengths on the metric graph, such that the difference of their geometric lengths lie in an interval that is allowed to shrink at a specific rate and positioned arbitrarily in the real line to obtain a uniform result. This result gives the same asymptotic if pairs of closed geodesics are ordered by their word length of the corresponding conjugacy class with respect to a free generating set of the fundamental group of the metric graph. In this work we followed the work done by Pollicott and sharp for counting pairs of closed geodesics in surfaces of negative curvature [1] and [2].

The techniques we use in our study include coding metric graphs by subshifts of finite type and the concepts from the thermodynamic formalism that appear in the ergodic theory of these systems. In particular, we use the spectral properties of transfer operators and their relationship to pressure and entropy [3], [4].

Keywords: Pairs of closed geodesics, metric graphs, asymptotic, word length, subshifts of finite type, thermodynamical formalism, transfer operator.

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Bulk-Boundary Eigenvalue Problem Governed by the (p, q)-Laplacian

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Abstract

In this paper we consider in a bounded domain $\Omega \subset \mathbb{R}^N$ an eigenvalue problem for the (p, q)-Laplacian plus some potentials, with the eigenvalue parameter both in the equation and the boundary condition. That arises from the study of double-phase parabolic equations under dynamical boundary conditions.

Under suitable assumptions, using the Nehari manifold method and a variational approach, we are able to determine the full eigenvalue set of this problem.

Keywords: Eigenvalues, (p, q)-Laplacian, Nehari manifold.

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Moment-Based Approximation for a Renewal Reward Process with Generalized Gamma Distributed Interference of Chance

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Abstract

In this study a new approximation method for ergodic distribution of a renewal reward process with generalized gamma distributed interference of chance is presented.

The generalized gamma distribution is a flexible probability distribution that extends the gamma distribution to include an additional shape parameter. The generalized gamma distribution can take the form of different distributions such as the Weibull, lognormal, exponential, Frechet and gamma depending on its parameters. Therefore, this study addresses a situation where the interference of chance random variables belongs to a very broad class of distributions.

The approximation method used in this study is inspired by the study of Kambo et. al. [2]. Our finding relies solely on the moments of demand random variables and the parameters of the generalized gamma distribution. In the literature, these processes have been studied with many different distribution families and asymptotic expansions have been obtained for different scenarios (see for example [3]). Unlike obtained asymptotic expansions, in this study the remaining terms are not implicitly represented with big Oh "O" or small oh "o" notations (which provide only upper bounds for remaining terms). Instead the remaining terms are presented explicitly in a functional manner, enabling an analysis of their convergence towards zero.

Keywords: Moment-based approximation, ergodic distribution, renewal reward process, generalized gamma distribution.

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Some Bounds for Several Statistical Indicators

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Abstract

In 1956, Aczél studied in [1], several methods in the theory of functional equations in one variable and showed the following inequality:

Let $\mathbf{a} = (a_1, a_2, ..., a_n)$ and $\mathbf{b} = (b_1, b_2, ..., b_n)$ be two sequences of positive real numbers such that

$$A^2 - a_1^2 - ... - a_n^2 > 0 \text{ and } B^2 - b_1^2 - ... - b_n^2 > 0,$$

where A and B are positive real numbers. Then

$$\left(A^2 - a_1^2 - ... - a_n^2\right)\left(B^2 - b_1^2 - ... - b_n^2\right) \leq \left(AB - a_1b_1 - ... - a_nb_n\right)^2,$$

with equality if and only if the sequences $\mathbf{a}$ and $\mathbf{b}$ are proportional, known as Aczél's inequality.

In this study, we define new bounds for the following statistical indicators: variance, standard deviation and coefficient of variation, using a refinement of Aczél inequality.

Keywords: Aczél inequality, statistical indicators, monotony of a sequence.

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Numerical Solution and Effective Error Estimation for a Mixed Problem for the Laplace Equation on a Rectangular Domain

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Abstract

A mixed boundary value problem for the Laplace equation is considered in a rectangular domain. When using the grid method for the numerical solution of this problem, the error estimate of the method contains the maximum modules of the derivatives of the sought solution. This naturally makes it difficult to use error estimates in practice. The discrete analogue of the Fourier method used in this work allows us to estimate the errors of the method only through the known data of the problem. This greatly simplifies the application of this assessment in practice.

Keywords: Numerical solution, Laplace equation, rectangular domain.

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Approximate Formulas for Moments of a Modification of Renewal-Reward Process in a Strip

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Abstract

A modification of the renewal-reward process in a strip is examined in this paper. First, we have proved that this process is ergodic under certain conditions, see [2]. Then, the exact formulas are obtained for the ergodic distribution and moments of the ergodic distribution of the process. Additionally, Laplace transform and Milne ratio, see [1] is used to study the asymptotic behavior of all moments of the ergodic distribution of this process. As a result of the analysis, the approximate formulas for the moments of the ergodic distribution of this process are presented. Finally, in order to show that the approximate formulas are close enough to the exact formulas, a special example is considered.

Keywords: Renewal-reward process in a strip, moments of the ergodic distribution, approximate formulas, Laplace transform.

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Energy Spectrum for Scarf-Grosche Potential

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Abstract

A quantum system with Scarf-Grosche potential [1, 2, 3] has been investigated analytically in non-relativistic theory. We report bound state solutions of the Schrödinger equation for the relevant potential using the functional analysis approach and the asymptotic iteration method [4, 5, 6]. First, eigenenergies and the corresponding eigenfunctions are derived from polynomial solutions of the Jacobi differential equation [7]. To double-check the analytical solutions, we adopt the asymptotic iteration method (AIM). We obtain the energy spectrum and the eigenfunction solutions which depend on hypergeometric functions within the framework of AIM. Therefore, the results are cross-checked in the present work. It is worthwhile to mention that the asymptotic iteration approach is a powerful and useful method in analyzing bound state solutions.

Keywords: Asymptotic iteration method, hypergeometric function, Jacobi polynomial.

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Solving Partial Differential Equations by α-Parameterized New Iterative Method

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Abstract

In this paper, the α-parametrized new iterative method with approximate analytical solutions is used to solve differential equations, which arise from the development of the New Iterative Method (NIM), by introducing a linear combination to the final formulation of NIM, This linear combination is in terms of the parameter α, which works to synthesize solutions and adjust them for the exact solution. The proposed method, referred to as (α-PNIM), demonstrates a reliable and efficient solution without any discrimination or restrictive assumptions by calculating Maximum Absolute Error (MAE) and Mean Square Errors (MSE) for an example set of linear and nonlinear partial differential equations to be solved in this paper.

Keywords: α-Paramitrized new iterative method, new iterative method, Dandelion optimizer, partial differential equations, meta heuristic.

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Solving a Mathematical Model of Diabetic Atherosclerosis by Numerical Methods

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Abstract

Atherosclerosis is one of the leading causes of death worldwide. Making the situation worse, current data on the prevalence of diabetes published by the International Diabetes Federation (IDF) shows that there are approximately seven million diabetic patients in the age range of 20-79 in Türkiye, which corresponds to approximately 15 of the total adult population. It increases inflammation associated with atherosclerosis. Patients with diabetes are twice as likely to have a heart attack or stroke. In this article, we discuss a simplified mathematical model for diabetic atherosclerosis that includes LDL, HDL, glucose, insulin, free radicals (ROS), β cells, macrophages, and foam cells. For this model we obtain a system of partial differential equations and then solve them by numerical methods.

Keywords: Atherosclerosis, partial differential equations, numerical methods, LDL, HDL.

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Investigating Tension B-Spline Functions: Advancements in Numerical Approximation Techniques

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Abstract

This study investigates the incorporation of tension B-spline functions into the collocation method framework for solving partial differential equations (PDEs), emphasizing their connection to approximation theory and special functions. Tension B-splines, well-known for their enhanced shape control and local deformation capabilities, offer a versatile approach to numerical approximation. In comparison to traditional B-splines, this work explores the improved representation abilities and processing efficiency of tension B-splines of higher degree. The study discusses tension B-splines and their advantages, specifically their adaptive nature and reduced overshoot. Additionally, it conducts theoretical investigations of stability and convergence and demonstrates their application in the collocation technique.

Keywords: Tension B-spline functions, partial differential equations, stability analysis.

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A New Analogue of the Appell-type Changhee Polynomials with Fibonomial Calculus

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Abstract

In this paper, the Appell-type Changhee F-polynomials, which correspond of the Appell-type Changhee polynomials in Fibonomial Calculus, are introduced. Furthermore, the Appell-type Fibo-Changhee matrix and the Appell-type Fibo-Changhee polynomial matrix are defined. Some relations and identities involving these polynomials and matrices are established.

Keywords: Changhee polynomials, Appell-type Changhee polynomials, generating function, generalized Pascal matrix, Fibonacci matrix.

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On Some Inequalities of Vector Functions

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Abstract

In this paper, we study vector functions of $\mathbb{R}^n$ into itself, which are of the form $x \mapsto f(|x|)x$, where $f : (0, \infty) \to (0, \infty)$ is continuous function and call these radial functions. For the case when $f(t) = t^c$ for some $c \in \mathbb{R}$, some upper bounds for the distance of image points under such radial functions are established. We also discuss some properties of the distortion function $\varphi_K(r)$ associated with the quasiconformal Schwarz lemma (see [3] for the quasiconformal Schwarz lemma). Moreover, we study how the distances in the hyperbolic and j-metrics are transformed under the radial mappings (see [2]).

Keywords: Vector functions, radial functions, hyperbolic metric.

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On Some Concave Generalizations of Diamond Alpha Bennett-Leindler Type Dynamic Inequalities

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Abstract

Diamond alpha Bennett-Leindler type dynamic inequalities are extended via concavity. By these inequalities, their special cases in delta and nabla settings as well as for the continuous and discrete scenarios, are unified. Moreover some of these particular results have appeared in the literature for the first time as well. If concavity condition is removed, these inequalities provide generalizations of diamond alpha Bennett-Leindler type dynamic inequalities [1].

Keywords: Diamond-alpha time scale calculus, Hardy-Copson inequality, Bennett-Leindler inequality, concavity.

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α-Analogue of Gamma Function

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Abstract

In this talk, we first present a new time scale, namely α-time scale [1] which is determined in terms of a weighted operator α. The α- operator is indeed a convex combination of delta and nabla shift operators of (q,h)-time scales. The α-time scale is a generic time scale which unifies and extends delta and nabla (q,h)-analysis as well as their reductions to delta/nabla h-and q-analysis. Based on [1, 2], we introduce the concepts of Taylor series, exponential function, power function, gamma function, their properties and applications on α-time scale.

Keywords: α-time scale, exponential function, power function, gamma function.

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A Modified Similarity Measure for Continuous Function Valued Intuitionistic Fuzzy Sets and An Application on Classification

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Abstract

The focus of this study is on the continuous function valued intuitionistic fuzzy sets (CFVIFSs) and fuzzy classification. The combination of CFVIFSs, which is an innovative approach, and classification, which is a fundamental machine learning technique, provides a different perspective in this study. Furthermore, we define a new similarity measure for CFVIFSs by modifying an existing similarity measure for the intuitionistic fuzzy sets from the literature. This similarity measure and the aggregation operator for CFVIFSs are used in the fuzzy classification and results are obtained by creating a new solution logic.

Keywords: Continuous function valued intuitionistic fuzzy sets, fuzzy classification.

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Integral Operator with Bivariate Mittag-Leffler Function with Respect to Function

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Abstract

Bivariate Mittag-Leffler functions are important in fractional calculus. By motivating series formula of integration operator ${}_c\mathcal{E}^{\delta;\omega_1,\omega_2}_{\alpha,\beta,\gamma}$ involving bivariate Mittag-Leffler function $E^{\delta}_{\alpha,\beta,\gamma}$ in the kernel, we generalize the theory by considering the new operator with respect to function. We emphasise the significance of the conjugation connections with the classical Riemann-Liouville fractional calculus to demonstrate numerious fundamental characteristic properties of this generated operator. Also, we illustrate some important results concerning the generated integral operator with respect to function, such as product rule and chain rule.

Keywords: Bivariate Mittag-Leffler function, fractional calculus, integral transforms, fractional integral equations, fractional calculus with respect to functions.

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A Generalized Correlated Random Walk Converging to Fractional Brownian Motion

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Abstract

We propose a new algorithm to generate a fractional Brownian motion, with a given Hurst parameter, $H \in [1/2, 1]$, using the correlated Bernoulli random variables with parameter p, having a certain density. This density is constructed using the link between the correlation of multivariate Gaussian random variables and the correlation of their dichotomized versions; and the relation between the correlation coefficient and the persistence parameter. We prove that the normalized sum of trajectories of the proposed random walk yields a Gaussian process whose scaling limit is the desired fractional Brownian motion. Please mainly refer to the studies [1, 2, 3, 4, 5, 6, 7].

Keywords: Correlated random walk, dichotomized variables, fractional Brownian motion, Gaussian process.

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Hyers-Ulam-Rassias Stability of Fractional Delay Differential Equations with Caputo Derivative

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Abstract

During this talk, we will study the Hyers-Ulam-Rassias (HUR) stability of a nonlinear Caputo fractional delay differential equation (CFrDDE) with multiple variable time delays. We obtain two new theorems with regard to HUR stability of the CFrDDE on bounded and unbounded intervals. The method of the proofs is based on the fixed point approach. The HUR stability results of this paper have indispensable contributions to theory of Ulam stabilities of CFrDDEs and some earlier results in the literature.

Keywords: Fractional differential equation, Caputo derivative, multiple variable delays, HUR stability, the alternative fixed point theorem.

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Orthogonal Polynomials and the Associated Jacobi Operator

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Abstract

To a sequence of orthonormal polynomials p_n on the real line is associated a Jacobi operator $(T, D(T))$, i.e., the operator in ℓ^2 defined as the closure of the Jacobi matrix acting on the subspace of complex sequences with only finitely many non-zero terms. It is well-known that it is symmetric with deficiency indices either $(0,0)$ or $(1,1)$. The two cases correspond to the moment problem behind being either determinate or indeterminate, i.e., there is exactly one orthogonality measure or there are several and then infinitely many orthogonality measures for p_n. In the determinate case $(T, D(T))$ is self-adjoint, but not in the indeterminate case, where it has infinitely many self-adjoint extensions. We shall focus on this case, and for a complex number z we let $\mathfrak{p}_z, \mathfrak{q}_z$ denote the sequences $(p_n(z))$ and $(q_n(z))$, where q_n denote the polynomials of the second kind. These sequence are known to be square summable.

It is known that $\mathfrak{p}_z, \mathfrak{q}_z \notin D(T)$ for all $z \in \mathbb{C}$. We determine whether linear combinations of $\mathfrak{p}_u, \mathfrak{p}_v, \mathfrak{q}_u, \mathfrak{q}_v$ for $u, v \in \mathbb{C}$ belong to $D(T)$ or to the domain of the self-adjoint extensions of T in ℓ^2. The results depend on the four Nevanlinna functions of two variables associated with the moment problem. We also show that $D(T)$ is the common range of an explicitly constructed family of bounded operators on ℓ^2. The talk is based on [1].

Keywords: Jacobi matrices and operators, indeterminate moment problems.

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Convergence and High Order of Approximation by Steklov Sampling Operators

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Abstract

It is well-known that one of the central results in the Sampling Theory, is provided by the celebrated Whittaker-Kotelnikov-Shannon (WKS) sampling theorem for the interpolation of band-limited, finite energy signals. Driven by the purpose to weaken the assumptions of such an important result, Butzer and his students started the theory of sampling-type series by the introduction of the so-called generalized sampling operators in the 1980's (see, e.g., [2]). These approximation operators were revealed to be very suitable for linear prediction and the reconstruction of continuous signals. Subsequently, in [1], Bardaro, Butzer, Stens and Vinti introduced a Kantorovich version of the generalized sampling operators that were very suitable in order to reconstruct not-necessarily continuous signals. Hence, such operators (in the multivariate form) brought back to the study of applications based on image reconstruction (see, e.g., [3, 5]). Recently, the previous sampling-type operators have been generalized and extended by the introduction of the so-called Steklov sampling operators ([4]). Here, the idea is to consider a sampling series based on a kernel function that is a discrete approximate identity, and which constitutes a reconstruction process of a given signal f, based on a family of sample values which are Steklov integrals of order r evaluated at the nodes k/w, $k \in \mathbb{Z}$, $w > 0$. The convergence properties of the introduced sampling operators in continuous functions spaces and in the L^p-setting have been studied. Additionally, the main properties of the Steklov-type functions have been exploited in order to establish results concerning the high order of approximation. Such results have been obtained in a quantitative version thanks to the use of the modulus of smoothness of the approximated function, and assuming suitable Strang-Fix conditions. Finally, comparing the proposed Steklov sampling series with the previously mentioned sampling-type operators, the effectiveness of the introduced constructive method of approximation has been demonstrated.

Keywords: Steklov sampling operators, high order of approximation, Strag-Fix type conditions.

Acknowledgments: The author has been partially supported within the project: PRIN 2022: "AI- and DIP-Enhanced DAta Augmentation for Remote Sensing of Soil Moisture and Forest Biomass (AIDA)" funded by the European Union under the Italian National Recovery and Resilience Plan (NRRP) of NextGenerationEU, under the Italian Ministry of University and Research (MUR) (Project Code: 20229FX3B9, CUP: J53D23000660001).

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Solving the Fractional Fredholm-Volterra Integrodifferential Equations by Morgan-Voyce Polynomials

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Abstract

In this study, Morgan-Voyce polynomials have been applied to construct an approximation method to obtain the solutions of the linear fractional Fredholm-Volterra integro-differential equations (IDEs). The fractional derivative is considered in conformable sense. By this approximation method, the fractional IDE has been transformed to a linear algebraic equations system with the aid of the collocation points. Additively, the matrix relation for the conformable fractional derivative of Morgan-Voyce polynomials is obtained for the first time in the literature. Furthermore, some numerical examples are represented to demonstrate the preciseness of the method.

Keywords: Conformable fractional derivative, Morgan-Voyce polynomials, Fredholm-Volterra integrodifferential equations.

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Trigonometric Krätzel Functions: Definition, Properties, and Applications

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Abstract

In this study, two new functions, the Sine Krätzel and the Cosine Krätzel functions, are examined. These functions are defined by combining classical trigonometric functions with Krätzel functions [1], [2]. In our work, the fundamental properties, definitions, and analyses of these new functions are presented in detail. The potential applications of the Sine Krätzel and Cosine Krätzel functions in mathematical analysis have been explored, and their relationships with various integral transforms and differential equations have been investigated.

Keywords: Krätzel functions, sine Krätzel function, cosine Krätzel function. integral transforms, special functions.

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Asymptotics of Orthogonal Polynomials With Respect to the Abel and Related Weights

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Abstract

The Abel weight function arises in integrals induced by the Abel-Plana summation formula, which makes the orthogonal polynomials relative to it important for numerical summation. Thus, for example, for numerical integration we need their zeros. We will show that these orthogonal polynomials, as well as orthogonal polynomials with respect to some related weight functions, admit a representation as oscillatory integrals. Then we will investigate their asymptotic behavior for n large and give asymptotic expansions. These asymptotic expansions will make it possible to approximately locate their zeros.

Keywords: Orthogonal polynomials, asymptotic, Abel weight.

Parametric q-Exponential Functions and q-Appell Polynomials

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Abstract

In this paper, we introduce parametric q-exponential functions which includes among others little, big and avarege q-exponential functions. Then we consider the new Parametric q-Appell polynomials corresponding to this q-exponential function and investigate their properties such as pure and q-derivative recurrence relation and determinantal representation. Finally, some special cases are introduced and their properties have been drawn directly form the main result.

Keywords: Exponential functions, Appell polynomials, difference equations.

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New Developments For The Jacobi Polynomials

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Abstract

In this work, we first develop a new and more general form of the Jacobi differential equation, and by means of its general solution we define the k-Jacobi polynomials and we obtain related generating functions and Rodrigues' formula. We also show its orthogonality and derive its norm. In the next two parts, by defining the the k-Gegenbauer and k-Legendre differential equations and the k-Gegenbauer and the k-Legendre polynomials corresponding to a their special solution, respectively, we achieve properties similar to those found for the k-Jacobi polynomials. Some new properties including explicit formula, generating functions and recurrence relations are also obtained for these polynomials. Furthermore, we derive various families of bilinear and bilateral generating functions and also display some examples.

Keywords: Jacobi polynomials, Gegenbauer polynomals, Legendre polynomials, hypergeometric function, orthogonal polynomials.

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Approximation by Matrix-Valued Functions via Summability Process

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Abstract

In this paper, using the notion of the $\mathcal{A}$-summation process, which includes both convergence and almost convergence, we obtain an approximation theorem for matrix-valued positive linear operators. In addition, we give an example that shows our theorem is more applicable than a classical one.

We also consider the rates of convergence of these operators.

Keywords: Matrix-valued functions, $\mathcal{A}$-summability, positive linear operators, Korovkin type theorem, rates of convergence.

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Asymptotic Formulas of Non-Self Adjoint Hill Operator with Trigonometric Potential

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Abstract

We will identify non-self adjoint Hill operators with different potentials

$$q(x) = ae^{-i2\pi x} + be^{i2\pi x} + ce^{-i4\pi x} + de^{i4\pi x}$$

and

$$p(x) = \alpha e^{-i2\pi x} + \beta e^{i2\pi x} + \gamma e^{-i4\pi x} + \delta e^{i4\pi x}.$$

Based on our previous study the spectra of are the same if $ab = \alpha\beta$, $cd = \gamma\delta$, $a^2d = \alpha^2\delta$ and $b^2c = \beta^2\gamma$ where $a, b, c, d, \alpha, \beta, \gamma$, and δ are complex numbers.

In this study we will express inverse result of the theorem that $ab + cd$ can be determined by a sequence of the eigenvalues of the operator $H_t(a, b, c, d)$ for some values of t.

Keywords: Isospectrality, Hill operator, trigonometric potential.

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Left-Definite System of Equations

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Abstract

In this talk, we will introduce a system of equations in a kind of left-definite form. Then we will try to explain how to employ the Prüfer angle method to get some information on the eigenvalues of the corresponding boundary-value problem containing a spectral parameter in the boundary conditions. Moreover, we will share some Frechet derivatives of the eigenfunctions of the problem.

Keywords: Left-definite equations, Prüfer angle method, Frechet derivatives.

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Construction and Error Analysis of a Gaussian Rule for Integrals Involving Bessel Functions

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Abstract

In this work we consider the approximation of integrals involving powers, exponentials and Bessel functions of the first kind by means of a Gaussian rule ([2]). The commonly used techniques for the computation of the three-term recursion of a family of orthogonal polynomials are the Chebychev and the modified Chebychev algorithms (see [3, Sect. 2.3]). However, these approaches may be inaccurate for growing number of quadrature points, because the problem is severely ill-conditioned (see e.g. [4]). We develop an alternative technique based on the fact that the recurrence coefficients can be computed by solving a linear system with the moment matrix. In this framework, since the weight function of the problem can be interpreted as a perturbation of the weight function of the generalized Laguerre polynomials, we use the moment matrix of these polynomials as preconditioner.

As for the estimate of the quadrature error of the Gaussian formula, we consider the generalized averaged Gaussian rule (see e.g. [5]), together with a tentative a-priori approximation of the error based on the theory of analytic functions ([1]). The numerical experiments reveal that the technique introduced for the computation of the recurrence coefficients is definitely more stable than the Chebychev and modified Chebychev algorithms. Moreover, the error approximation given by the averaged Gaussian rule appears to be very accurate. Finally, the heuristic but quite effective a-priori estimate potentially allows the introduction of a truncated approach, to reduce the number of function evaluations.

Keywords: Gaussian quadrature, Bessel function of the first type, Generalized averaged Gauss rule.

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On One Boundary Value Problem with Symmetric Potential

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Abstract

In recent years, interest has been opened at dependent spectral parameter in the boundary conditions for ordinary differential equation. This problems are usually derived from partial differential equations and studied by a lot of researchers [1]–[4].

In this work, we deal with a boundary value problem including integrable and symmetric potential in the related interval and eigenvalues in the boundary conditions. We calculate the eigenvalues of the boundary value problem, asymptotically.

Keywords: Integrable potential, boundary value problem, asymptotic estimates.

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Reconstruction of the North Atlantic Double-Gyre Circulation with Genetic Programming

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Abstract

In this study, Genetic Programming is explored as a data-driven approach to reconstruct eddy-resolved simulations of the North Atlantic double-gyre. Stemming from Genetic Algorithms, Genetic Programming is a method of symbolic regression which can be used to extract temporal or spatial functionalities from simulation snapshots [1, 2]. The North Atlantic double-gyre is simulated by a stratified quasi-geostrophic model which is solved using high-resolution CABARET scheme [3, 4]. The statistically stationary simulations of the double-gyre model are considered for 100 years after a 100-year spin-up period. The simulation results are compressed using classic Proper Orthogonal Decomposition (POD) and Spectral Proper Orthogonal Decomposition (SPOD) [5] which allows separating the spatial modes according to frequency to characterise spatiotemporal coherent structures in mesoscale oceanic turbulence. The time variant coefficients of the two reduced-order models are next fed into a Genetic Programming code for reconstruction. Finally, the parameter space of objective functions in Genetic Programming is explored to capture the key statistical properties of the original time series such as variance and auto-correlation function.

Keywords: Genetic programming, North Atlantic double-gyre, machine learning, turbulence.

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Some Results of Korovkin Type for Nonlinear Operators

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Abstract

In this talk we give Korovkin type theorems for nonlinear operators by using power series summability method. Also we consider an example showing that our results are general than classical ones.

Keywords: Korovkin theorem, nonlinear operator, sublinear operator.

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Euler Method Extended via Fluctuationlessness Theorem

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Abstract

The general expression of the Fluctuationlessness Theorem [1, 2, 3] states that the matrix representation of an algebraic operator which multiplies its argument by a scalar univariate function, is identical to the image of the independant variable's matrix representation over the same subspace via the same basis set [4], under that univariate function, when the fluctuation terms are ignored. Just by using this basic idea, this principle applied on the remainder term of a Taylor expansion a highly versatile approximation can be obtained. Taking into consideration that Euler method and higher order Taylor methods are using Taylor expansion to solve IVP problems [5], we applied the Fluctuationlessness theorem to the remainder term of Taylor expansion expressed in integral form and adapted it to Euler method by extending it.

Keywords: Fluctuationlessness Approximation, Matrix Representation of a Function, Remainder Term of Taylor Expansion, Euler Method, IVP problems

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New Generalization of Baskakov-Kantorovich Operators for Better Error Estimation

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Abstract

In this paper, we present a new family of generalized Baskakov-Kantorovich operators and examine their approximation properties. We compare these new operators with the classical Baskakov-Kantorovich operators, showing that the new ones perform better approximation results. Additionally, we study the uniform convergence and the rate of convergence of these operators using the first and second order modulus of continuity. Finally, we provide numerical examples and graphs to illustrate the results of our study.

Keywords: Baskakov operators, Baskakov-Kantorovich operators, polynomial approximation, rate of convergence,modulus of continuity, uniform convergence.

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An Estimate for Durrmeyer type Exponential Sampling Series in (Mellin) Orlicz Spaces

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Abstract

We obtain a quantitative estimate for the Durrmeyer type exponential sampling series by using the concept of modulus of smoothness given in [1] with the help of a suitable modular functional defined on Orlicz spaces which are also reduced versions of Mellin-Orlicz spaces. Then we customize this estimate for several examples of Orlicz spaces such as $L^p_\mu(\mathbb{R}^+)$-spaces.

Keywords: Durrmeyer type exponential sampling series, Orlicz spaces, modulus of smoothness.

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A New Family of Finite Biorthogonal Polynomials in Two Variables

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Abstract

In this paper, we define three novel classes of finite biorthogonal polynomials in two-variables with the help of a new method enabling construct new bivariate biorthogonal polynomials and obtain their some properties. In addition, we investigate limit and transition relations with some known and new families.

Keywords: Finite biorthogonal polynomials, bivariate biorthogonal polynomials, orthogonal polynomials.

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Riesz-Bessel Transforms of High Order on Weighted Variable Lebesgue Spaces

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Abstract

The theory of harmonic analysis consists of significant operators such as singular integrals, maximal operators, sharp maximal operators, Riesz potentials, convolution type operators and approximate identities. On various function spaces, the boundedness of these operators and their versions which are generated by Laplace-Bessel differential operator are considerable problems of this theory.

In this talk, by using generalized Hardy type operators, we have obtained that Riesz-Bessel transforms of high order $R_{\gamma}^{(k)}$ are bounded on weighted variable Lebesgue spaces $L_{p(\cdot),\omega,\gamma}(\mathbb{R}_{k,+}^{n})$.

Keywords: Hardy type operator, singular integral operator, weighted variable Lebesgue space.

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Sequences of m-Term Deviations in Hilbert Space

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Abstract

Let D be a dictionary in a Hilbert space H, that is, a set of unit elements whose linear combinations are dense in H. We consider the least m-term deviation $\sigma_m(x)$ of an element $x \in H$: this is the distance from x to the set of all m-term linear combinations of elements of D. We present a dichotomy result: for any dictionary D, either the sequence $\{\sigma_m(x)\}_{m=0}^{\infty}$ decreases exponentially for every $x \in H$, or the rate of convergence $\sigma_m(x) \to 0$ can be arbitrarily slow. We also seek universal dictionaries realizing all strictly decreasing null sequences as sequences of m-term deviations. All commonly used dictionaries turn out not to be universal. In particular, the least rational deviations in Hardy space H^2 do not form certain strictly monotone null sequences. There are no universal dictionaries in finite dimensional Hilbert spaces, but in every infinite dimensional Hilbert space a universal dictionary can be constructed.

Keywords: Hilbert space, m-term approximation, dictionary, deviations, rational approximation.

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Demi-*ab* Continuous Operators and Their Properties

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Abstract

The concepts of compact and continuous operators hold significant importance in Banach lattice theory. These operator classes can also be generalized to more abstract structures such as topological vector lattices. In 1966, the concepts of demicontinuous and demicompact operators were introduced by W. V. Petryshn, based on norm convergence, to construct and investigate the structure of fixed point sets for nonlinear operators acting on Hilbert and Banach spaces. However, in vector lattices, there are various types of convergence in addition to norm convergence.

In recent years, extensive research has been conducted on unbounded convergences. In this talk, we will define classes of demi-*ab* continuous operators based on various types of a and b convergences on Banach lattices. Alongside classical concepts of order and norm convergence, unbounded convergences such as unbounded order, unbounded norm, and unbounded absolute weak convergence, which are defined in vector lattices, will form an integral part of this study. Moreover, the relationships between *ab* continuous operators and demi-*ab* continuous operators will be elucidated. The results of this study are based on the article [1].

Keywords: Banach lattice, demi operators, convergence, order convergence, relatively uniformly convergence, unbounded order convergence, unbounded norm convergence, unbounded absolutely weak convergence.

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Korovkin-Type Approximation Theorems for Functions with the Help of $\mathcal{I}$-Statistical Convergence

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Abstract

In this study, by means of the concept of $\mathcal{I}$-statistical convergence, we prove a general Korovkin-type approximation theorem for sequences of positive linear operators from $H_\omega(K)$ to $C_b(K)$, where $K = [0, \infty)$. Later, an example is given such that our new approximation result provides but its classical and statistical cases do not work. We compute the rates of convergence for the double sequences of positive linear operators by using the modulus of smoothness.

Keywords: Statistical convergence, $\mathcal{I}$-statistical convergence, Korovkin-type approximation theorem.

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Parametric Extensions of a Certain Family of Bernstein-Type Rational Functions

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Abstract

In this talk, we introduce a parametric extension of a certain family of Bernstein-type rational functions on the interval $[0, \infty)$. Firstly, we give the construction of these operators and evaluate the test functions which will be used in computing their central moments. Then, we investigate the weighted approximation properties. Next, shape preserving properties of these operators are studied. Finally, the convergence properties of these operators to some functions are illustrated by graphics. This work is motivated by the papers [1, 2, 3].

Keywords: Bernstein-type rational functions, Voronovskaja type theorem.

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Binomial Transform of the Bivariate Fibonacci Quaternion Polynomial Sequences and Some Properties

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Abstract

The major aim of this paper is to deal with binomial transform for Bivariate Fibonacci Quaternion Polynomial sequence. We give new formulas for recurrence relation, generating function, Binet formula and some identities for the binomial sequence. Furthermore, we obtain various kinds of sums for these quaternion polynomials. Working with bivariate Fibonacci quaternion polynomial sequence, we have found the most general formula that includes all binomial transform with recurrence relation from the second order. In the last part, matrix representations are derived for the relevant binomial sequence.
Keywords: Bivariate Fibonacci quaternion polynomial, binomial transform, generating function, Binet formula.

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Numerical Approximation of Fractional Volterra Integral Equations via Kantorovich Operators

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Abstract

This paper presents a novel approach to the numerical approximation of fractional Volterra integral equations, encompassing both those of the first and second kinds, through the utilization of Kantorovich operators. The proposed methodology is meticulously analyzed for convergence, ensuring its theoretical soundness and robustness. To validate the effectiveness and practical utility of this approach, an extensive series of numerical experiments is conducted. These experiments are designed to rigorously test and illustrate the method's performance. Comparative analyses are then performed, highlighting the strengths and potential advantages of our method over existing techniques. The results demonstrate the approach's validity and practicality, providing compelling evidence of its capability to solve fractional Volterra integral equations accurately and efficiently.

Keywords: Fractional Volterra integral equations, Kantorovich operators, convergence analysis.

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Approximation Properties of Some Modifications of Bernstein Operators

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Abstract

This presentation explores the approximation properties of the Stancu variant generalized Bernstein operators. We analyze various moment estimates and investigate several local direct approximation theorems. Additionally, we delve into further approximation features of these newly defined operators, including the Voronovskaya-type asymptotic theorem and pointwise estimates. Through graphical and numerical comparisons with known linear positive operators, we demonstrate superior approximation results in terms of convergence behavior, computational efficiency, and consistency. Finally, we apply these operators to obtain a numerical solution for a special case of the fractional Volterra integral equation of the second kind.

Keywords: Pointwise estimates, computational efficiency, direct approximation theorems.

Acknowledgment: This study is supported by Van Yüzüncü Yıl University Scientific Research Projects Coordination Unit under the grant number FBA 2023-10867.

Piecewise Generated Frames

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Abstract

In this presentation, we introduce frames that are generated from a bounded lineer operator F on a Hilbert space H together with scalars $\{a_i, b_i\}_{i\in I}$. We show that every frame $\{x_i\}_{i\in I}$ in H can be written piecewise with a bounded lineer operator F on H and some scalars $\{a_i, b_i\}_{i\in I}$ but converse of the statement is not true in general. We give several classifications of piecewsie frames for different bounded lineer operators F and scalars $\{a_i, b_i\}_{i\in I}$ in H.

Keywords: Frame vectors, piecewise frames, phase and norm retrieval.

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Modelling of Count Data in Circular Statistics

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Abstract

This study focuses on the development and evaluation of circular Poisson regression models tailored for circular count data. The research objectives include the formulation of specialized Poisson regression models respecting the circular nature of the data, comprehensive evaluation of model performance through simulation studies and real-world applications, exploration of practical applications in fields like ecology and meteorology, and the provision of implementation guidelines for researchers and practitioners. By achieving these objectives, this research contributes to advancing the understanding of circular statistics and provides valuable insights into estimating counts on circular scales accurately.

Keywords: Poisson regression, count data, circular data, linear-circular regression, circular poisson regression model.

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Boundedness of Riesz Potential on Homogeneous Variable Exponent Herz-Morrey-Hardy Spaces

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Abstract

In harmonic analysis, studies of inequalities of Riesz potential in various function spaces have a very important place. Variable exponent Morrey type spaces and the examines of the boundedness of such operators on these spaces have an important place in harmonic analysis and have become an interesting field. In this work, we consider the boundedness of Riesz potential on homogeneous variable exponent Herz-Morrey-Hardy spaces under some conditions.

Keywords: Riesz potential, variable exponent, homogeneous Morrey-Herz-Hardy space.

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On a New Perspective for the Mellin-Lebesgue Spaces

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Abstract

In this talk, we express the first modulus of smoothness of a function f in the Mellin-Lebesgue space. Thanks to this, we state the rate of convergence. Later, some pointwise convergence results are obtained for the Mellin-Gauss-Weierstrass operators. In the last part, we have the pointwise convergence of them at any Lebesgue point of a function f. Studies related to the subject can be found in [1]–[7].

Keywords: Weighted Lebesgue spaces, pointwise convergence, quantitative estimate.

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Poisson's Equation for Borel Ergodic Theorems

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Abstract

The solution of Poisson's equation $y = (I - T)x$, for a given linear operator T on a Banach space X and a given $y \in X$, can be obtained in several ways. In [1, 2], this solution is discussed with iterative methods while in [3, 4], the proof is examined via ergodic theory. We will be interested in the method by Lin and Sine dealing with the convergence of $x_n := \frac{1}{n}\sum_{k=1}^{n}\sum_{j=0}^{k-1} T^j y$ to the solution x, for a given mean ergodic operator T. Replacing Cesàro means by a nonnegative regular matrix $A = (a_{nk})$ the same problem has also been considered in [5]. In this talk, replacing Cesàro means by Borel summation method which is a sequence to function method, we study weak convergence of $x_t := e^{-t}\sum_{j=0}^{\infty}\frac{t^j}{j!}\sum_{k=1}^{j} T^{k-1}y$ in order to solve the equation $y = (I - T)x$.

Keywords: Ergodic Theorems, Poisson's Equation, Borel Ergodic theorem, bounded linear operator

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Computation of Generalized Modulus of an n-gon

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Abstract

In this talk, we analyze the Schwarz-Christoffel (SC) transformation for polygons and introduce the concept of a generalized modulus. The function exhibits many interesting properties and allows for the algorithmic solution of the SC parameter problems and enables the approximation of conformal modulus. It is known that when n vertices are removed from the complex plane, they form conformally equivalent classes. These classes are called the modulus of the manifold and have a degree of $n-3$. Therefore, between two triangles, there always exists a conformal mapping, but for quadrilaterals, there is one extra parameter called the conformal modulus. A conformal mapping exists between two polygons if and only if their conformal moduli are equal.

In our previous work [1], we studied these problems for quadrilaterals, completely solving the SC (Steklov–Courant) parameter problem and developing an algorithm for approximating the conformal modulus of the quadrilateral. We introduce $\varphi(x_1,\dots,x_{n-3};\tau_1,\dots\tau_{n-1})$ for the real positive variables x_i, $i=\overline{1,(n-3)}$, and real positive numbers $\tau_1,\dots,\tau_{n-1}>0$ that satisfy the condition $\tau_1+\dots+\tau_{n-1}<n-2$. The generalized modulus φ is defined using Lauricella functions and has many interesting properties, like measuring the ratio between adjacent side lengths.

Acknowledgments: The research supported by GNSF as part of grant no. FR22-354, titled "Problem of factorization and invariants of holomorphic bundles on Riemann surfaces."

Keywords: Schwarz–Christoffel transformation, conformal modulus, Lauricella functions.

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Extremal Polynomials on the Real Line

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Abstract

In this talk, we will discuss the theory for weighted Chebyshev polynomials and orthogonal polynomials on the real line. The weighted Chebyshev polynomials T_n are the monic polynomials that minimize the weighted sup-norm on a given set and monic orthogonal polynomials P_n minimize the L_2 norm associated with a Borel measure.

We will survey results involving the lower bounds of the norms of the extremal polynomials mentioned above. We will also discuss some recent results on asymptotics for the lower bounds, which generalize some classical results in Widom's seminal paper [1]. These recent results are based on a joint work in progress with Maxim Zinchenko (University of New Mexico).

Keywords: Widom factors, Chebyshev polynomials, orthogonal polynomials.

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On a Generalized Concept of Balancing Numbers

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Abstract

Since the first research on balancing numbers was made, many researchers have been interested in these numbers. Many studies have been done regarding these numbers, including some properties that these numbers provide, several generalizations of the concept of these numbers, the relationships between these numbers and the other integer sequences, and the construction of new integer sequences using the Diophantine equation of these numbers. In this study, a generalization of the concept of balancing numbers that is made by adding the third balancing number to the left side of the Diophantine equation of balancing numbers is considered. Properties of the integer sequence that is obtained as a result of this generalization is examined. Furthermore, several identities regarding this sequence are derived. Besides these, the functions that generating these numbers were presented.

Keywords: Balancing numbers, diophantine equation, generating functions.

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On a Generalization of the Sequence of Gaussian Balancing Numbers

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Abstract

In this study, a generalization of the sequence of Gaussian balancing numbers called as the sequence of Gaussian k-balancing numbers is introduced. Thus, the concept of k-balancing numbers is extented to the complex plane. Within the scope of this research, we first introduce the notion of Gaussian k-balancing numbers. Then we present some of the properties of this sequence such as recurrence relation, Binet formula and generating function. Furthermore, we obtain a variety of sum formulas associated with these numbers.

Keywords: Recurrence relation, generating function, sum formulas.

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A Wavelet Type Generalization of Bernstein Operators

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Abstract

In this talk, motivated by [4], we introduce a wavelet type generalization of Bernstein operators, which depends on a non-negative real parameter. We find some Bernstein-Markov type inequalities and investigate some L^p-approximation $(1 \leq p \leq \infty)$ results for the new operators.

Keywords: Bernstein polynomial, wavelets, Bernstein-Markov type inequalities.

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Evolution of Biological Patterns on Annular Domains

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Abstract

In this talk, a domain-dependent stability analysis for a reaction-cross-diffusion system is presented to understand the role of geometry and cross-diffusion in the evolution of the biological pattern formation. Bifurcation analysis of the system is accomplished for the cross-diffusive reaction-diffusion model. We derive conditions on the domain size for an annulus and generate parameter spaces associated with Turing difusion-driven instability, Hopf and trascritical instabilities. To support theoretical findings, finite element numerical simulations showing the pattern formation on non-convex geometries are presented. We illustrate finite element simulations that reveal spatial and spatiotemporal patterns in the dynamics of a cross-diffusive system. These observed patterns resemble which found in ring-shaped cross-sectional profiles of hypoxic tumours an annulus.

Keywords: Reaction-diffusion systems, cross-diffusion, pattern formation, parameter spaces, spatiotemporal dynamics, annular region.

References:

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On Some Properties of a Class of Integral Operators Preserving Exponential Functions on the Half-Line

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Abstract

In this talk, various approximation properties of integral operator sequences which are defined as a particular class in weighted exponential Lebesgue spaces will be investigated. These operators were previously considered in [5] in the space $C^*[0,\infty)$. In the current study, modulus of continuity will be used and quantitative theorems will be given. In particular, quantitative theorems will be proved with the help of the Peetre-K functional. Research consistency will be presented for cases where the kernel is specifically selected and discussed. Results will be supported by numerical calculations and graphical representations by comparing with older versions of the operators.

Keywords: Integral operators, positive linear operators, modulus of continuity.

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A Robust Study on Atangana-Baleanu Fractional Coupled Burgers Equation

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Abstract

This study focuses on the novel Atangana-Baleanu method employed to obtain new numerical solutions for the Coupled Burgers Equation with Atangana-Baleanu fractional derivatives. The ABq-EHATM is used to numerically solve this system. This paper presents an investigation of the numerical solutions obtained using this method. Furthermore, the proposed method has been subjected to numerical simulation using Maple software, considering different fractional orders.

Keywords: Burgers equation, Atangana-Baleanu q-Elzaki homotopy analysis transform method, Atangana-Baleanu Elzaki transform.

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A Novel Study on Caputo-Fabrizio Fractional Cahn-Allen Equation

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Abstract

This study focuses on the novel Caputo-Fabirizo method employed to obtain new numerical solutions for the Cahn-Allen equation with Caputo-Fabirizo fractional derivatives. The CFq-EHATM is employed to numerically solve this equation. An analysis of the numerical solutions found using this method is presented. In addition, the suggested approach has undergone numerical simulation with various fractional orders, employing Maple software.

Keywords: Cahn-Allen equationl, Caputo-Fabrizio q-Elzaki homotopy analysis transform method, Caputo-Fabrizio Elzaki transform.

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A New Study on Caputo-Fabrizio Fractional Smoking Model

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Abstract

Smoking is a widespread social phenomenon observed globally, especially at educational institutions and during important occasions. According to the World Health Organization, smoking is classified as the primary avoidable factor leading to illness and the third leading cause of mortality in humans. To explore this subject, this work focuses on studying the dynamics of the fractional order quitting smoking model using the Caputo-Fabrizio differential operator. The numerical solution of the model is obtained using the Caputo-Fabrizio q-Elzaki homotopy analysis transform method (CFq-EHATM). A comparison of the numerical solutions obtained using this method is provided. Furthermore, the proposed scheme has been subjected to numerical simulation using different fractional orders, utilizing Maple. The numerical findings have been validated through the use of illustrative images. The simulation demonstrates the suitability of the model under consideration.

Keywords: Smoking model, Caputo-Fabrizio fractional operator, Elzaki transform.

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A Novel Study on Atangana-Baleanu Fractional Benney Equation

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Abstract

This study focuses on the novel Atangana-Baleanu method employed to obtain new numerical solutions for the Benney Equation with Atangana-Baleanu fractional derivatives. The ABq-EHATM is used to numerically solve this equation. This paper presents an investigation of the numerical solutions obtained using this technique.Furthermore, the proposed method has been subjected to numerical simulation using Maple software, considering different fractional orders.

Keywords: Benney equation, Atangana-Baleanu q-Elzaki homotopy analysis transform method, Atangana-Baleanu Elzaki transform.

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Approximation Properties of Generalized q-Favard-Szász-Mirakjan Operators of Max-Product Kind

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Abstract

It has already been shown that linear structure is not the only one which allow us to construct approximation operators. By using maximum instead of sum in usual linear operators, maximum-product kind operators have been introduced and it is the starting point of nonlinear approximation. Many linear operators have been considered from this perspective and some approximation results of these operators have been obtained. Furthermore, there are also studies dealing with the approximation properties of these operators by q-calculus but very rare.

In this talk, we present the degree of approximation for generalized q-Favard-Szász-Mirakjan operators of max-product kind by using a Shisha-Mond type theorem.

Keywords: q-Analysis, nonlinear operators, rate of convergence.

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Stability Analysis of an SAIR Epidemic Model with Logistic Growth in Susceptible Compartment

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Abstract

SAIR (Susceptible-Asymptomatic-Infected-Recovered) model is a compartmental model used in epidemiology to understand and predict the spread of infectious diseases within a population acknowledging that not all infected individuals exhibit symptoms, yet they can still transmit the disease to others [1]. We aim in this work to enrich the understanding of epidemic dynamics by employing an SAIR model while supposing the susceptible compartment follows logistic growth.

Stability and bifurcation analysis are essential tools in understanding the behavior of epidemic models [2]. Employing mathematical analysis and numerical simulations, we explore the stability and bifurcation characteristics of the proposed compartmental SAIR epidemic model [3]. Our study encompasses the examination of equilibrium points, their stability under various parameter conditions, and the identification of potential bifurcation phenomena.

Furthermore, we examine the implications of our findings for disease control strategies. By uncovering the existence of multi-stable states within the model dynamics, we highlight the significance of taking proactive measures to prevent the initial spread of infection. Hence, we aim to contribute to the advancement of epidemic modeling and provide valuable guidance for policymakers and healthcare professionals in their efforts to combat infectious diseases and safeguard public health.

Keywords: SAIR epidemic model, logistic growth, bifurcation analysis.

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On Asymptotic Properties and Quantitative Results of Wavelet Type Operators

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Abstract

This paper deals with construction and studying wavelet type linear approximation operators by using the compactly supported Daubechies wavelets related to the target function f. We also study some asymptotic properties, quantitative and Voronovskaya-type results of the newly introduced wavelet type operators.

Unlike Fourier analysis (sinusoidal functions), wavelets can be tuned and adapted to the signals and are inherently local. Since there is no single wavelet, they can be designed to suit individual applications. They are ideal for adaptive systems or operators that adjust themselves to match the function. In other words, wavelet expansion, or reconstruction via wavelets, allows for more accurate local identification and separation of signal features.

We note that the results obtained for wavelet-type operators defined using some special cases of wavelets represent a natural extension of at least the classical operators and their Kantorovich-type modifications and are closely related to quasi-interpolation operators.

Keywords: Wavelets, compactly supported Daubechies wavelets, linear approximation process, asymptotic approximation.

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Zagreb Indices of a Discrete Topological Graph Generated by a Set of 4 Elements

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Abstract

In this talk, we will see how to obtain a discrete topological graph G generated by a set with 4 elements; $X = \{1, 2, 3, 4\}$ (see [1]). Then we mention some properties of this graph and calculate the first and second Zagreb indices of the graph G (see [2]), defined as

$$M_1(G) = \sum_{uv \in E(G)} \big[d(u) + d(v)\big] \quad \text{and} \quad M_2(G) = \sum_{uv \in E(G)} \big[d(u).d(v)\big],$$

where u and v are vertices, uv is an edge of G. Furthermore, $d(u)$ and $d(v)$ stand for the degree of u and v, respectively.

Keywords: Discrete topological graph, Zagreb indices.

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Asymptotic Expansion of Wavelet Type Generalized Bézier Operators

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Abstract

In approximation theory, Bernstein polynomials and their Bézier variants are well-known. Since the discrete type positive linear operators cannot be used for L^p $(1 \leq p < \infty)$ approximation. In order to obtain some positive results for approximating these functions by means of the discrete type operators their Kantorovich and Durrmeyer modifications are considered.

By using the effects and relations between different function spaces provided by the wavelets we will propose a generalization and extension of the theory of approximation by introducing an integral operator, called wavelet type operators [1, 2, 3, 4, 5]. Very recently, wavelet type generalized Bézier operators [6] obtained and studied by using the compactly supported Daubechies wavelets of the given function f. The main purpose of this study is to investigate the asymptotic expansion and to give some convergence results for these operators.

Keywords: Bézier basis, wavelets, asymptotic expansion.

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Positive Linear Operators and Convex Stochastic Orders

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Abstract

The inequalities involving positive linear operators and convex functions can be expressed in terms of convex stochastic orders. We review some known results, provide new ones and present some open problems. Here is a typical example. Let V_n be the Baskakov operators (corresponding to the negative binomial distribution) and S_n the Szász-Mirakyan operators (associated with the Poisson distribution). If a convex function $f \in C[0,\infty)$ is in the domain of S_n and in the domain of V_n, then

$$V_n f \geq S_n f, \ \ n \in \mathbb{N}. \tag{1}$$

Conversely, if $f \in C^2[0,\infty)$ satisfies (1), then f is convex. The problem is to prove that if $f \in C[0,\infty)$ satisfies (1), then f is convex.

Keywords: positive linear operators; convex functions; convex stochastic orders.

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Second-Order Differential Inclusions with Two Small Parameters

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Abstract

Consider in a real Hilbert space H the following problem

$$\begin{cases} -\varepsilon u''(t) + \mu u'(t) + Au(t) + Bu(t) \ni f(t), \ 0 < t < T, \\ u(0) = u_0, \quad u'(T) = 0, \end{cases} \tag{$P_{\varepsilon\mu}$}$$

where $T > 0$ is a given time instant, $\varepsilon > 0$, $\mu \geq 0$ are small parameters, $A : D(A) \subset H \to H$ is a maximal monotone operator (possibly multivalued), and $B : H \to H$ is a Lipschitz operator (or monotone and Lipschitz on bounded sets). Consider also the following reduced problem, denoted by (P_μ),

$$\begin{cases} \mu u'(t) + Au(t) + Bu(t) \ni f(t), \ 0 < t < T, \\ u(0) = u_0, \end{cases}$$

where $\mu > 0$, as well as the algebraic equation (inclusion)

$$Au(t) + Bu(t) \ni f(t), \ \ 0 \leq t \leq T. \tag{E_{00}}$$

We investigate existence and uniqueness of solutions to the above problems and to equation (E_{00}), as well as continuity of the solution to problem $(P_{\varepsilon\mu})$ with respect to $\varepsilon > 0$ and $\mu \geq 0$. Moving forward, we are also interested in the convergence of the solution of problem $(P_{\varepsilon\mu})$ to the solution of problem $(P_{\mu 0})$, as $\varepsilon \to 0_+$ and $\mu \to \mu_0$, where μ_0 is a fixed positive number, as well as the convergence of the solution of problem $(P_{\varepsilon\mu})$ to the solution of the equation $Au + Bu \ni f(t)$ as $\varepsilon \to 0_+$ and $\mu \to 0_+$. Last, but not least, we investigate applications in areas such as the regularization of the nonlinear heat equation, or the regularization of the telegraph system.

Keywords: Lions regularization, maximal monotone operator, Lipschitz operator, approximation, heat equation, telegraph differential system

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On Biorthogonal Bivariate Jacobi Konhauser Polynomials

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Abstract

Recently, Özarslan and Elidemir (2023) introduced a methodology for constructing two-variable biorthogonal polynomial families with the help of one-variable biorthogonal and orthogonal polynomial families. The primary objective of the paper is to introduce novel class of two-variable biorthogonal polynomials namely bivariate Jacobi Konhauser polynomials. We investigate several fundamental properties of these polynomials including their biorthogonality property, operational formula, generating function, and integral representation. Furthermore, we investigate their images under the Laplace transform, fractional integral and derivative operators. Corresponding to these polynomials, we define the new type bivariate Jacobi Konhauser Mittag Leffler (JKML) functions and obtain the similar properties for them. Finally, we introduce an integral operator containing the bivariate JKML functions in the kernel.

Keywords: Biorthogonal polynomials, bivariate Mittag Leffler function, fractional calculus.

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Weighted Fractional Inequalities For An Abstract Convexity

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Abstract

Fractional calculus is a popular field nowadays. There are numerous generalization of the integral operators [1]. Thus, the mathematical expressions are generalized accordingly.

Abstract convexity is one of the sub field of convexity. There are various abstract convexity types. Inequalities are the most known applications of convex functions [2, 3]. In this study, inequalities via weighted fractional integral operator for $\mathbb{B}$-convex functions are given. Besides, the generalization of the results are proved. Some applications of the obtained inequalities are shown.

Keywords: Convexity, fractional integrals, $\mathbb{B}$-convexity, weighted fractional.

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Approximation by Kantorovich-type Max-Min Neural Network Operators and Applications to Signal Processing

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Abstract

This study deals with maximum-minimum neural network operators activated by sigmoidal functions [1]. Our aim is to construct Kantorovich variant of max-min neural network operators. We obtain approximation theorems in L^p-spaces. Via the K-functionals (see [2]), we give some quantitative estimates for L^p-convergence. Then, we illustrate our examples. Finally we apply our approximations for denoising in biomedical signals and compare their performances with other kinds of neural network operators.

Acknowledgement: This study is supported by the Scientific and Technological Research Council of Türkiye (TÜBİTAK).

Keywords: L^p-convergence, Kantorovich operators, neural network operators, pseudo-linear operators, signal processing.

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Summation of Some Bilateral Hypergeometric Series via Lagrangian Inversion

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Abstract

Several well-known summations of the Gaussian hypergeometric series ${}_2F_1(a,b;c;x)$ may be obtained from Lagrange's inversion formula. One of these is originally due to Newton and another was found by Stirling. I will explain how they arise as special cases of a formula which evaluates in closed form the bilateral series ${}_2H_2(a,b;c,d;x)$ when its parameters are such as to satisfy $a+b=2c-1$ and $d-c=1/2$. This formula is itself the special case $r=2$ of a more general formula which, for any $r\geq 2$, expresses a particular ${}_rH_r$ series with three free parameters in terms of the solutions of a polynomial equation of degree r. I will describe how these formulae relate to general theorems on the factorization of hypergeometric and basic hypergeometric series.

Keywords: Hypergeometric series, summation formulae.

Power Means of Random Variables and Characterizations of Distributions via Fractional Calculus

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Abstract

This talk will be based on the preprint [1]. We investigate fractional moments and expectations of power means of complex-valued random variables by using fractional calculus. We deal with both negative and positive orders of the fractional derivatives. The one-dimensional distributions are characterized in terms of the fractional moments without any moment assumptions. We explicitly compute the expectations of the power means for both the univariate Cauchy distribution and the Poincaré distribution on the upper-half plane. We show that for these distributions the expectations are invariant with respect to the sample size and the value of the power.

Keywords: Fractional moments, power mean, characterization of distribution, Cauchy distribution, Poincaré distribution.

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[1] K. Okamura and Y. Otobe, Power means of random variables and characterizations of distributions via fractional calculus, preprint, arXiv 2312.02698

Some Explicit Formulas for Appell Polynomials

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Abstract

We prove that the nth polynomial of a sequence of polynomials $(A_k(x))_{k\in\mathbb{N}}$ satisfying the conditions $A_0'(x) = 0$ and $A_n'(x) = nA_{n-1}(x)$ for $n \geq 1$ can be written uniquely as a linear combination of the polynomials $x^n, (x+1)^n, \ldots, (x+mr)^n$ for any fixed integer m greater than or equal to n and for any given complex r. Interesting results on this subject can be found for example in [1, 2, 3].

Keywords: Appell polynomials, Bernoulli Polynomials, Euler polynomials.

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On Different Summable Sequence Spaces

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Abstract

For a perfect scalar-valued sequence space Λ, we deal with several topological properties of the space $\Lambda(E)$ of Λ-summable sequences in a locally convex (or a Banach) space E, such as barrelledness, distinguishedness and reflexivity. As these notions pass through the continuous dual of $\Lambda(E)$, we are led to deal also with the space $\Lambda[E]$ (resp. $\Lambda\langle E\rangle$) of weakly (respectively strongly) Λ-summable sequences in E. Each of these spaces is equipped with an appropriate Hausdorff locally convex topology, so that we can investigate the (quasi-) barrelledness, the distinguishedness and the (semi-) reflexivity in $\Lambda(E)$, in terms of the same properties in both E and Λ, and the AK property. Here Λ is equipped with an arbitrary polar normal topology. A new notion of distinguishedness is introduced and studied. An example separating it with the classical one is provided. Our outcomes complete, extend and generalize some known results.

Keywords: Summable sequence space, barrelled space, distinguished space, reflexive space.

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On Numerical Solution of Quasilinear Integral–Algebraic Equations

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Abstract

In this report we consider a system of Volterra integral equations of the form

$$A(t)x(t) + \int_0^t K(t,\tau,x(\tau))d\tau = f(t), 0 \le \tau \le t \le 1, \tag{1}$$

where matrix $A(t)$ is a given $(n \times n)$-matrix, $K(.) : \Omega \to R^n, \Omega \subseteq R^{n+2}$, $f(t)$ is a given, $x(t)$ is an unknown n-dimensional vector function. It is assumed that

$$detA(t) \equiv 0 \tag{2}$$

and the elements of $A(t)$, $K(t,\tau,x(\tau))$, $f(t)$ have the smoothness necessary for all calculations. Systems (1) with condition (2) are commonly referred to integral-algebraic equations (IAEs). There are not so many works devoted to the numerical solution of linear IAEs and research into the construction of numerical methods for quasilinear problems (1) are unknown to the authors. Conditions for the existence of a unique sufficiently smooth solution have been obtained in terms of matrix pencil [1]. An algorithm for their numerical solution is proposed, which is based on the simplest quadrature formula and linearization of a nonlinear integrand. Illustrative examples and results of numerical calculations of test examples are given.

Keywords: Volterra integral equation,integral-algebraic equations, matrix pencil, numerical method.

Acknowledgments: The research was supported by the grant from the Russian Science Foundation (project No 22-11-00173).

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Approximation with Szász-Chlodowsky Operators Employing Hermite-Modified Laguerre Polynomials

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Abstract

This article explores a Chlodowsky-type extension of Szász operators [6] using the Hermite-modified Laguerre polynomials [2]. The convergence properties of these operators are established by employing the universal Korovkin-type property, and the order of approximation is determined using the classical modulus of continuity. Additionally, the weighted $\mathfrak{B}$-statistical convergence and statistically weighted $\mathfrak{B}$-summability properties of the operators are derived. Theoretical results are supported by numerical and graphical examples.

Keywords: Szász operators, Hermite-modified Laguerre polynomials, order of convergence, modulus of continuity, weighted $\mathfrak{B}$-statistical convergence

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Approximation Properties of Sampling Kantorovich Operators in Sobolev Settings

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Abstract

In this talk, we will analyze some approximation results of Kantorovich-type sampling operators in the context of classical and fractional Sobolev spaces settings. In particular, in the fractional case, we will show some approximation properties for a new class of Sobolev spaces based on the well-known Gagliardo fractional Sobolev spaces and a class of recently introducted Sobolev spaces by Feng and Sutton.

Keywords: Sampling Kantorovich operators, approximation results, Sobolev spaces.

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Some Results for a General Class of Szász-Mirakjan-Durrmeyer Operators

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Abstract

We consider sequences of positive linear operators $S_{n,j}$ which can be viewed as a generalization of the Szász-Mirakjan-Durrmeyer operators [1], Phillips operators [2] and corresponding Kantorovich modifications of higher order.

In our talk we present commutativity results for the operators, their commutativity with certain differential operators and some spectral properties.

Keywords: Positive linear operators, Szász-Mirakjan-Durrmeyer operators, Phillips operators, Kantorovich operators.

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Nonlinear Sampling Kantorovich Operators and Applications to the Case of Exponential Sampling

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Abstract

In this talk, we present some recent results concerning the *nonlinear sampling Kantorovich operators* within the general framework of modular spaces $L^\rho(\mathbb{R})$ (see [5, 6]). In particular, we provide some quantitative estimates in such spaces, achieved through a direct approach, using a suitable definition of the modulus of smoothness with respect to the modular ρ. This extends the field of possible applications and enables us to give a unifying approach for approximation problems in several settings. In fact, modular spaces include Musielak-Orlicz spaces, which contain, for instance, weighted-Orlicz spaces and Orlicz spaces, as well as spaces of functions equipped by modulars that are not of integral type. Here, also the case of the approximation in Lebesgue and weighted-Lebesgue spaces is included. A wide literature can be found in [3, 7, 8, 9].

Furthermore, based on the recent interest in the study of exponential sampling series (see, e.g. [1, 2, 4]), we investigate the convergence properties of the *exponential version* of the nonlinear sampling Kantorovich operators. We establish convergence results in spaces of log-uniformly continuous and bounded functions, as well as in Orlicz spaces. The basic notions and tools of Mellin Analysis play a crucial role in this investigation, proving to be the most suitable approach to address the aforementioned approximation problems.

Our study is motivated by the practical applications of nonlinear operators in signal processing. Notably, these operators are suitable for describing signals generated by seismic events or explosions, and nonlinear transformations generated by signals that, during their filtering process, produce new frequencies.

Keywords: Nonlinear sampling Kantorovich operators, modular spaces, Orlicz spaces, modulus of smoothness, order of approximation, nonlinear exponential sampling operators.

Acknowledgments: The second author (Mariarosaria Natale) has been partially supported within the project PRIN 2022: "AI- and DIP-Enhanced DAta Augmentation for Remote Sensing of Soil Moisture and Forest Biomass (AIDA)" funded by the European Union under theItalian National Recovery and Resilience Plan (NRRP) of NextGenerationEU, under the Italian Ministry of University and Research (MUR) (Project Code: 20229FX3B9, CUP: J53D23000660001).

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The Multiple Jain-Appell Polynomials

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Abstract

In the present paper, we introduce the multiple Jain-Appell polynomials and investigate convergence properties of these operators by using Korovkin's theorem. The error of approximation of these operators has been given by using various modulus continuities in weighted space. Local rate of approximation has been obtained with the second modulus of continuity and Lipschtiz class functions [3]. Finally, Voronovskaya type asymptotic approximation has been given for Multiple Jain-Appell operators.

Keywords: Szász-Mirakyan operators, multiple Appell operators, Jain-Appell operators, rate of Convergence, Lipschitz type function space.

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Voronovskaja Type Results for Modified Bernstein Operators

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Abstract

In this study, we derive new Voronovskaja-type asymptotic formulas for modified Bernstein operators based on regular summability methods. We also give several applications of our results, including Cesàro, Riesz, Abel, and Borel summability methods. Additionally, we explore analogous findings for the Kantorovich version of the operators.

Keywords: Bernstein polynomials, Bernstein-Kantorovich polynomials, Voronovskaja-type asymtotic formula, regular summability methods.

Acknowledgments: This research was conducted as a part of the Ph.D. thesis under the supervision of Professor Oktay Duman.

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A New Representation of the Generalized k-Bivariate Mittag-Leffler Function

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Abstract

In this paper, we will present a new formala of the generalized k-bivariate Mittag-Leffler function. We investigate some elementary properties of the newly bivariate Mittag-Leffler function as like as the hypergeometric function representation and we explore the relationship with the Wright function. Also, the Hankle representation and double Mellin integral are presented. Further, we apply the general transform to the new function and we prove some results of fractional calculus.

Keywords: Bivariate Mittag-Leffler function, special functions, fractional calculus, general transform.

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Regularization and Approximation Results for Durrmeyer Sampling Operators in L^p-Spaces

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Abstract

In this talk, we present some recent regularization and approximation results for a semi-discrete family of sampling operators, namely *Durrmeyer sampling operators* (DSO) [2], within the L^p-setting ($1 \leq p \leq +\infty$). This study belongs to the theory of sampling series, which holds a central position in the field of Approximation Theory due to its widespread applications, especially in Signal and Image Processing. Notably, DSO, as double convolution operators based on two distinct kernels (a discrete and a continuous one), expand other well-known families of sampling operators, including both generalized [3] and Kantorovich [1, 7] types.

Specifically, this talk focuses on a recent investigation into the regularization properties of DSO within L^p-spaces, employing a distributional approach [4]. In [5], we prove that the regularization by DSO turns out to be strongly influenced by the regularity of the discrete kernel. Here, we show some *regularization results*, considering such kernel belonging to different functional spaces, including Sobolev spaces. Moreover, this study has been preparatory in achieving both direct and inverse approximation results [6]. In particular, direct results allow us to face the problem of the *higher order of approximation* by DSO, via finer quantitative estimates in terms of the modulus of smoothness of the function. We also show the qualitative version of such results. Meanwhile, the *inversion problem* is highly delicate and leads to a complete characterization of the well-known generalized Lischphitz classes in terms of the order of convergence of DSO in L^p-setting.

The talk concludes by discussing the main results via specific instances of kernels.

Keywords: Durrmeyer sampling type operators, regularization, Sobolev spaces, modulus of smoothness, higher order of approximation, inverse problem.

Acknowledgments: The second author (Michele Piconi) has been partially supported within the project: PRIN 2022 PNRR: "RETINA: REmote sensing daTa INversion with multivariate functional modeling for essential climAte variables characterization", funded by the European Union under the Italian National Recovery and Resilience Plan (NRRP) of NextGenerationEU, under the MUR (Project Code: P20229SH29, CUP: J53D23015950001).

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On Integral-Algebraic Equations with Variable Limits of Integration

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Abstract

The report presents results on the study of the existence and uniqueness of a sufficiently smooth solution to two problems:

$$A(t)x(t) + \int_{t-c}^{t} K(t,\tau)x(\tau)d\tau = f(t),\ t \in [0,1] \tag{1}$$

and

$$B(t)y(t) + \int_{at}^{t} L(t,\tau)y(\tau)d\tau = g(t),\ t \in [0,1], \tag{2}$$

where $A(t)$, $B(t)$, $K(t,\tau)$, $L(t,\tau)$ are given $(n \times n)$-matrices, $f(t)$, $g(t)$ are given, $x(t)$, $y(t)$ are unknown n-dimensional vector-functions. It is assumed that $c > 0$ and $x(t) = x_0(t)$, when $t \in [-c, 0)$, for problem (1). For problem (2), we suppose that $0 \leq a < 1$, where a is a scalar.

In terms of matrix pencils and solvability conditions for a system of linear algebraic equations, the conditions for the existence of a unique solution to the problems under consideration are given. We use the method of steps (for problem (1)) and the known results from functional analysis to prove this fact.

Also we present results on numerical solution to some classes of problems (1), (2).

Keywords: Volterra integral equation, matrix pencil, numerical method.

Acknowledgments: The research was supported by the grant from the Russian Science Foundation (project No 22-11-00173).

Discrete Null Field Equation Methods for Solving Laplace's Equation

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Abstract

Consider Dirichlet problems of Laplace's equation in a bounded simply-connected domain S, and use the null field equation (NFE) of the Green formulation, where the source nodes are located on a pseudo-boundary Γ_R outside of S. Simple algorithms are proposed in this paper by using the central rule for the NFE, and the normal derivatives u_ν of the solutions on the boundary Γ $(= \partial S)$ can be easily obtained. These algorithms are called the discrete null field equation method (DNFEM), because the collocation equations are, in fact, the direct discrete form of the NFE. It is discovered that the bounds of condition number are alike those by the method of fundamental solutions (MFS), to have the exponential growth as the number of unknowns increases. The numerical solutions u_ν also have the polynomial convergence rates as those by boundary element method (BEM). The errors and the stability of the DNFEM are explored in this paper, with comparisons to the MFS. One trouble of the DNFEM is the near singularity of integrations for the solutions in boundary layers. To handle this trouble, we develop two kinds of new techniques: I. The interplant techniques by Taylor's formulas with piecewise q-order polynomials and Fourier series, and II. The mini-rules of integrals, such as the mini-Simpson and the mini-Gaussian rule. Error analysis is made for Technique I to achieve optimal convergence rates. Numerical experiments are carried out for disk and peanut-like domains, to support the analysis of errors and stability. The numerical performance of the DNFEM is excellent for disk domains, to compete with the MFS. The errors with $O(10^{-4})$ can be obtained, which are satisfactory for most engineering problems. The traditional BEM suffers from the same trouble. For the solutions in boundary layers, the new techniques in this paper are simple without the sinh-transformation, they are also a contribution to the BEM. The materials of this talk are adapted from [1].

Keywords: Null field equation, discrete null field equation method, interior boundary layers, exterior boundary layers, method of fundamental solutions, Laplace's equations, stability analysis, error analysis, degenerate scales, pseudo-boundaries, sensitivity index.

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Topological Properties of Neutrosophic Operator in Sequence Spaces

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Abstract

This research explores the use of neutrosophic normed spaces to analyze the convergence of sequences of neutrosophic operators. By employing the tribonacci matrix, we illustrate the convergence characteristics of these operators. Additionally, we establish the properties of convergent sequences of neutrosophic operators within neutrosophic normed spaces. The theoretical findings are substantiated through numerical examples, demonstrating the validity and applicability of our conclusions.

Keywords: Sequence spaces, tribonacci matrix, neutrosophic sequence spaces, neutrosophic normed spaces.

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Special Affine Stockwell Transform: Theory, Applications and Uncertainty Principles

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Abstract

In this paper, we study the convolution structure in the special affine Fourier transform domain to combine the advantages of the well known special affine Fourier and Stockwell transforms into a novel integral transform coined as special affine Stockwell transform and investigate the associated constant Q-property in the joint time-frequency domain. The preliminary analysis encompasses the derivation of the fundamental properties, Rayleigh's energy theorem, inversion formula and range theorem. Besides, we also derive a direct relationship between the recently introduced special affine scaled Wigner distribution and the proposed SAST. Towards the culmination of this paper, we establish Heisenberg's uncertainty principle, logarithmic uncertainty principle and Nazarov's uncertainty principle associated with the proposed SAST.

Keywords: Stockwell transform, special affine Fourier transform, special affine scaled Wigner distribution, time-frequency analysis, uncertainty principle.

On Orthogonal Polynomials with a Generating Function of Rainville Type

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Abstract

We investigate polynomial sets, $\{P_n\}_{n\geq 0}$, with generating power series of Rainville type given by $A(t)F(xtA(t)-R(t))$, and satisfying the 2-order recursion

$$xP_n(x) = P_{n+1}(x) + \beta_n P_n(x) + \omega_n P_{n-1}(x),$$

where $n \geq 0$, $P_0(x) = 1$, $P_{-1}(x) = 0$, and $\{\beta_n\}$, $\{\omega_n\}$ are complex sequences. We give the relations between $\{\beta_n\}$, $\{\omega_n\}$ and the coefficients of the formal power series $A(t)$, $F(t)$ and $R(t)$. As an application, we give all the generating functions of the form $A(t)F(xtA(t) - R(t))$ for the (monic) ultraspherical, Hermite, and Chebyshev polynomials of the first and second kind.

Keywords: Generating functions, orthogonal polynomials, recurrence relations.

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On a Connection Between an Integrodifference System and Orthogonal Polynomials

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Abstract

In some biological situations, the spreading speed of a nonlinear model is governed by its linearization at low densities. We consider here, a non-autonomous integrodifference system (NIS) describing the demography and dispersal of a population in a one-dimensional habitat. It is shown that solutions of the linear integrodifference system (LIS) associated to the NIS at low densities can be represented by orthogonal polynomials. This allows us to express the upper bound of the speed stated in [1] by the nth root of these polynomials. On the other hand, inspired by [3], we use the asymptotic behaviour of a class of these polynomials and the saddle point method to give the asymptotic behaviour of the LIS solutions and speed's formula which coincides with the upper bound.

Keywords: Orthogonal polynomials, integrodifference equations, spreading speed.

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An Accurate Method for Solving Fractional Riccati Systems Based on the Operational Matrix Method

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Abstract

In this paper, we investigate the solution of fractional systems of Riccati equations. This system is important as it appears in several applications in science, such as control theory. We develop an iterative operational matrix method to solve this system. This modification helps us find the coefficients of the approximate solution iteratively, reducing the computational time and cost of the operational matrix method. Additionally, the new version makes this method easier to use and more practical without the limitations of the classical operational matrix method. We present several theoretical results, such as stability and uniform convergence of the proposed method. We test the numerical efficiency of the proposed method through several examples and comparisons with other researchers. Results show that the new method is promising and can be used for several applications

Keywords: Riccati system, operational matrix method, fractional calculus.

On the Global Solutions for the System of the Klein-Gordon Equation in Anti de Sitter Spacetime

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Abstract

In this contribution, we consider the Cauchy problem for the system of the Klein-Gordon equations in anti-de Sitter spacetime

$$\partial_t^2\Phi + n\Phi_t - e^{2t}\Delta\Phi + M\Phi = F(\Phi), \qquad (x,t) \in \mathbb{R}^n \times \mathbb{R},$$

$$\Phi(x,0) = \varphi_0(x), \qquad \partial_t\Phi(x,0) = \varphi_1(x), \qquad x \in \mathbb{R}^n,$$

where φ_0, φ_1 are in Sobolev space $W^{[n/2]+1,2}(\mathbb{R}^n.)$ Here, M is called the mass matrix which is real and diagonalizable matrix. In [1], Yagdjian and Galstian constructed the fundemantal solution of the Klein-Gordon equation in anti-de Sitter spacetime. Next, $L^p - L^q$ decay estimates were derived in [2]. In de Sitter spacetime, Yagdjian [3] proved global existence of the solutions for the system of semilinear Klein-Gordon equations. In this study, we investigate the global existence of the solutions to the system of semilinear Klein-Gordon equations in anti-de Sitter spacetime with the suitable conditon for $F = F(\Phi)$ by using the L^∞ estimate .

Keywords: Anti-de Sitter spacetime, Klein-Gordon equations, global solutions.

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A Novel Generalization of Brass-Stancu Operators

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Abstract

In this talk, we consider the operators defined by Brass [3] and Stancu [4]. We generalize the Kantorovich-type modification of these operators obtained in [2]. We delve into the L^p approximation properties of these new generalized operators. Moreover, we provide some estimates related to the rate of convergence.

Keywords: Stancu-type operators, L^p-convergence, averaged modulus of smoothness.

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Certain New Integral Formulas Involving the Generalized Multiindex Bessel Functions

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Abstract

In the present study, we establish some generalized integral formulas involving product of the multiindex Bessel function and other polynomials, which are expressed in terms of the Fox-Wright function. Further, ongiving suitable special values to the coefficient in the main results give many known classical polynomials. Therefore, we observe that our main results can lead to yield numerous other interesting integrals involving various Bessel functions and some known polynomials by suitable specialization of arbitrary parameters in the main theorems.

Keywords: Multiindex Bessel function, Fox-Wright function, Oberhettinger integral formula.

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A New Example of P-Ideals

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Abstract

In 1981, Freedman and Sember [1] defined the concept of the natural density and stated that the natural density has some of the properties of a measure but, in particular, is not a countably additive function. Therefore, certain additivity properties (called the properties (AP)), which are approximations to countable additivity, are of interest for some natural densities.

In 2000, Kostyrko, Šalát and Wilczynski [4] introduced the notion of $\mathcal{I}$-convergence of sequences of real numbers where the ideal $\mathcal{I}$ is a family of subsets of $\mathbb{N}$ satisfying certain conditions. They gave a similar of the property (AP) within the context of $\mathcal{I}$. The ideals with the property (AP) are called P-ideals.

We recently have introduced the concept of A-density with the rate of $o(a_n)$ and presented some basic properties of this concept where $A = (a_{nk})$ is a nonnegative regular matrix and $a = (a_n)$ is a positive nonincreasing sequence (see [3])

The class

$$\mathcal{I}_{\delta_{A,a}} = \left\{ E \subset \mathbb{N} : \delta_{A,a}(E) := \lim_{n\to\infty} \frac{1}{a_n} \sum_{k\in E} a_{nk} = 0 \right\}$$

form an ideal which is called the ideal of $A-$density zero with the rate of $o(a_n)$. In this talk, we obtain that $\mathcal{I}_{\delta_{A,a}}$ is a P-ideal.

Keywords: Density, ideal, P-ideal.

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The Generalization of the α-Meyer-König-Zeller Operators by Generating Functions

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Abstract

In this paper, we introduce generating functions type α-Meyer-König and Zeller operators. We prove a new Korovkin type theorem by using appropriate auxiliary test function. Secondly, we compute the rate of approximation by means of the modulus of continuity, Peetreís K-functional and modified Lipchitz class functionals. Also, we introduce the rth order generalization of these operators and explore their approximation properties. Finally, we introduce the particular form of the operators and study their approximation properties by obtaining functional partial differential equation which helps us calculate the moments easily.

Keywords: Positive linear operators, Meyer-König-Zeller operators, modified Lipschitz class.

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α-Phillips Operators

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Abstract

In this talk, we introduce α-Phillips operators. We study pointwise and weighted approximation properties of these operators. Also, we illustrate some graphs to show the convergence properties of the operators to certain functions. This work is motivated by the following references: [1, 2].

Keywords: Phillips operators, weighted approximation, pointwise approximation.

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Cubature Methods on Scattered Data by Different Kinds of Adaptive Interpolation

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Abstract

We construct cubature methods on scattered data via resampling on the support of known algebraic cubature formulas by different kinds of adaptive interpolation (polynomial [1], RBF [2], PUM [3]). This approach gives a promising alternative to other recent methods, such as direct meshless cubature by RBF, which is based directly on LOOCV optimized RBF integration over polygons [4] or least-squares cubature formulas, based on ℓ^2 weight minimization under polynomial moment matching conditions [5].

Keywords: Scattered data, numerical cubature, algebraic formulas, adaptive interpolation, RBF, PUM, multinode Shepard method.

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Estimation of a Finite Population Mean Under Random Non-Response Using Improved Nadaraya-Watson Kernel Weights

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Abstract

Non-response is a potential source of errors in sample surveys. It introduces bias and large variance in the estimation of finite population parameters. Regression models have been recognized as one of the techniques of reducing bias and variance due to random non-response using auxiliary data. In this study, it is assumed that random non-response occurs in the survey variable in the second stage of cluster sampling and that full auxiliary information is available throughout. Auxiliary information is used at the estimation stage via a regression model to address the problem of random non-response. In particular, auxiliary information is used via an improved Nadaraya-Watson kernel regression technique to compensate for random non-response. The asymptotic bias and mean squared error of the estimator proposed are derived. Besides, a simulation study conducted indicates that the proposed estimator has smaller values of the bias and smaller mean squared error values compared to existing estimators of finite population mean. The proposed estimator is also shown to have tighter confidence interval lengths at 95% coverage rate. The results obtained in this study are useful for instance in choosing efficient estimators of finite population mean in demographic sample surveys.

Keywords: Mean squared error, bias, kernel regression, two-stage cluster sampling, confidence interval lengths.

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Generalization of Sheffer-λ Polynomials

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Abstract

In this study, new generalization of Sheffer-λ polynomials are introduced by using the monomiality principle formalism and operational methods. Quasi-monomiality properties, differential equation, determinant representation and some other properties have been obtained for this family of polynomials. In addition, subpolynomial families of these polynomials were examined and their corresponding properties were obtained.

Keywords: Sheffer-λ polynomials, monomiality principle, differential equation.

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Quasi-Involutions of the Riordan Group

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Abstract

A Riordan array is an infinite lower triangular matrix, constructed from two formal power series. A Riordan quasi-involution is an aerated Riordan matrix whose inverse contains the exact same entries with $\pm$ signs on alternating non-zero subdiagonals [1, 2]. In this talk, we present special generating functions that give rise to a Riordan quasi-involution. We discuss about the quasi-involutions as combinatorial and algebraic objects in Riordan array research [4].

Expanding the concept of a quasi-involution to k-leveled aerated matrices, for $k > 1$, we analyse these elements that satisfy the quasi-involution property. From a combinatorial point of view, we present structural properties of these elements. We link them to known Riordan subgroups, and by introducing the theory of quasi-compressions, we prove a factorization theorem for a certain family of Riordan quasi-involution. Finally, we discuss the importance of quasi-involutions in the Heisenberg-Weyl algebra [3].

Keywords: Riordan arrays, generating functions, Riordan group, quasi-involutions.

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On $\mathcal{I}_2$-$\alpha\beta$-Statistical Relative Uniform Convergence and an Application to Approximation Theory

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Abstract

This article introduces an interesting type of convergence called $\mathcal{I}$-$\alpha\beta$-statistical relative uniform convergence for double sequences of functions. It has been introduced by us for the first time. Then, the approximation theorem of the Korovkin-type via this new type of convergence has been proved and an example has been given to show that the new type of convergence is stronger than before. Finally, the rate of $\mathcal{I}_2$-$\alpha\beta$-statistical relative uniform convergence has been computed.

Keywords: Korovkin type theorem, positive linear operator, $\mathcal{I}_2$-$\alpha\beta$-statistical relative convergence.

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Modern Problems in Quantum Optimal Control

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Abstract

In this presentation, we consider modern aspects of optimal control theory in quantum systems [1]. The connection between gradient-based optimization algorithms and pontryagin maximum principle will be shortly described. We will set a quantum control problem for a three-level quantum system [2] whose dynamics are governed by the Schrodinger equation and will formulate this problem using Pontryagin Maximum Principle [3]. We consider the existence of optimal solutions for this optimal control problem.

Besides, we present Brachistochrone, as the most important problem in optimal control theory, for quantum systems in this talk. Quantum brachistochrone is a quantum analogue of classical brachistochrone [4]. We review the basics of the quantum brachistochrone formalism and give some examples.

Keywords: Quantum control, quantum brachistochrone, pontryagin maximum principle, three-level quantum system, gradient-based optimization.

Acknowledgements: This work was supported by grant N FR 22-354 from the Shota Rustaveli National Science Foundation.

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Completely Monotone Invariance of Smoothing via Central Vector Lattice Differences

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Abstract

Using central difference expansions of summations, we study properties of a graduation (smoothing) process in the settings of Abelian semigroups with an involution and Dedekind complete vector lattices. We show that a-completely monotone functions, a variant of G. Choquet's completely monotone functions, are left fixed by graduations satisfying certain positivity conditions. Several numerical and theoretical examples and detailed discussions of various cases are given.

The theory of real completely monotone sequences can be found in [4] (see also [2]). The present work uses a-complete monotonicity, which is abstracted from [4, Sec. III.4]. G. Choquet's definition of complete monotonicity can be found in [3, Sec. III] and also in [1, Sec. 4.6] and it is for semigroups without involution.

Let $a \in S$ be a nonzero element of S, where S is an additive Abelian semigroup with an involution $^*\colon S \to S$, such that $a^* = -a$ for every $a \in S$. Let $W = E^S$, where E is a real Dedekind complete vector lattice, such as $L_p(\mu)$ for $1 \leq p < \infty$. A vector $\phi \in W$ is said to be a-completely monotone if $\phi \geq 0$ and $\nabla_a^k \phi \geq 0$ for every positive integer k.

Suppose that $t = (t_0, \ldots, t_M)$ satisfies $t_j \geq 0$ for $j = 0, 1, \ldots, M$. We prove that if ϕ is a-completely monotone then $\mathcal{T}_{a,t,m}(\phi)$ is also a-completely monotone where $\mathcal{T}_{a,t,m}$ is the graduation process

$$\mathcal{T}_{a,t,m}(\phi) = t_0\phi(s) + t_1([3]_a - [1]_a)\phi(s) + \cdots + t_m([2m+1]_a - [2m-1]_a)\phi(s)$$

for all $s \in S$, where $[2m+1]_a$ is summation by $(2m+1)s$.

Keywords: Vector lattice, smoothing, positive operator, central difference, completely monotone.

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A Hybrid Variable Selection Approach Using Binary Particle Swarm Optimization and Elastic Net Regularization in High-Dimensional Data

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Abstract

Variable selection in high-dimensional data is an essential step to increase the prediction performance of machine learning methods. In the case of high-dimensional data, having more variables than the number of observations negatively affects machine learning methods in terms of calculation efficiency and prediction performance. In this study, a new two-stage hybrid variable selection approach is proposed, combining two methods, wrapper and embedded, for variable selection in high-dimensional data. In the first stage of the proposed method, the binary particle swarm optimization (BPSO) approach as the wrapper method was used to reduce the number of variables due to its many advantages such as simplicity, efficiency, and fast convergence [1]. However, BPSO does not guarantee an optimal solution like other metaheuristic algorithms. To solve this problem, the Elastic Net (EN) approach as the embedded method was used as the second stage of the proposed hybrid approach to improve the variable selection process because of some advantages such as flexibility and time efficiency in selecting penalty parameters [2]. The performance of the proposed approach in variable selection was compared with the BPSO and EN approaches based on the simulation studies under different settings according to the mean sum of squared errors (MSSE) for coefficients, precision, recall, specificity, accuracy, and F1-score metrics. The proposed hybrid approach was also applied to the real data set and the prediction performance of the proposed approach was compared with the BPSO and EN approaches. According to the simulation and real data set results, the proposed hybrid method showed better variable selection and prediction performances than other compared methods.

Keywords: Variable selection, hybrid approach, binary particle swarm optimization, elastic net, high dimensional data.

References:

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Complex Generalized Stancu Operators Depending on Three Parameters

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Abstract

In this talk, we consider a new generalization of complex Stancu operators. We obtain quantitative upper estimates for the convergence, lower estimates from a qualitative Voronovskaja-type theorem and then the exact degree of simultaneous approximation by these operators attached to analytic functions in a disk centered at the origin with radius greater than 1. Also, we give some graphical and numerical examples.

Keywords: Perturbed Bernstein operator, complex perturbed Bernstein-type operator, Stancu operator, simultaneous approximation, equivalence.

References:

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A New Approach for Solving Minimax Problems Using New Generation Smoothing Techniques

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Abstract

The main objective of this work is to address finite minimax problems. A novel approach is proposed to reformulate minimax problems using indicator functions. An investigation is conducted to explore the connections between the original problem and the reformulated problem. A newly developed smoothing technique is proposed by approximating the indicator functions, in light of the revised formulation of minimax problems. A new algorithm has been developed to address the reformulated and smoothed problems. Finally, the performance of the algorithm is demonstrated on several test problems, and comparison of the obtained numerical results with alternative approaches is provided.

Keywords: Minimax problem, non-smooth optimization, smoothing technique.

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Approximation Properties of Szász-Chlodowsky Operators with Truncated Exponential-Gould-Hopper Polynomials

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Abstract

This article investigates a Chlodowsky-type extension of Szász operators [6] utilizing truncated exponential-Gould-Hopper polynomials [2]. The convergence properties of these operators are demonstrated using the universal Korovkin-type property, and the order of approximation is assessed through the classical modulus of continuity. Furthermore, the weighted $\mathfrak{B}$-statistical convergence and statistically weighted $\mathfrak{B}$-summability properties of the operators are established. Theoretical findings are corroborated with numerical and graphical examples.

Keywords: Szász operators, Truncated exponential-Gould-hopper polynomials, order of convergence, modulus of continuity, weighted $\mathfrak{B}$-statistical convergence

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Existence and Uniqueness of the Multirational h-Blossom

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Abstract

Blossoming is a powerful technique used to study properties of the Bernstein basis functions of positive degree and the corresponding Bézier curves and surfaces with these blending functions [1]. The blossom of a degree n polynomial $P(t)$ is the unique, symmetric, multiaffine function $p(u_1, \ldots, u_n)$ that reduces to $P(x)$ along the diagonal. But if we are interested in rational functions or Bernstein basis functions of negative degree or arbitrary analytic functions rather than polynomial functions, then we need to consider a different blossoming scheme. This kind of blossom is called the multirational blossom [2]. The multirational blossom of order k and degree $-n$ of a k-times differentiable function $f(t)$ defined as a multivariate function $f(u_1, \ldots, u_k/v_1, \ldots, v_{n+k})$ characterized by four axioms: bisymmetry in the u and v parameters, multiaffine in the u parameters, satisfies a cancellation property and reduces to $f(t)$ along the diagonal. The existence of this blossom is proved in [2]. However, the uniqueness is also crucial for several identities and properties of the multirational blossom. The uniqueness of the multirational blossom is established in [3].

Now we generalize the multirational blossom to h-calculus and introduce axiomatically what we call the multirational h-blossom. We aim to establish the existence and uniqueness of the multirational h-blossom. To do so, we first provide an explicit formula which allows us to calculate the multirational h-blossom of any continuous function. Then we provide a constructive proof of the uniqueness of the multirational h-blossom. Finally we briefly mention why this blossom matters.

Keywords: Multirational blossom, h-calculus, divided difference.

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Is the Limit q-Durrmeyer Operator Shape-Preserving?

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Abstract

In this talk, the question in the title is answered in the affirmative. Specifically, the shape-preserving properties of the limit q-Durrmeyer operator, $0 < q < 1$, introduced by V. Gupta in [3] has been considered. It has been proved that if a function is m-convex along $\{q^j\}_{j=0}^{\infty}$, then its image under the limit q-Durrmeyer operator D_q remains m-convex along $\{xq^j\}_{j=0}^{\infty}$, $x \in [0,1]$. Furthermore, this operator is both monotonicity- and convexity-preserving. In addition, it turns out that, for each monomial $e_m(x) = x^m$, $x \in [0,1]$, $m \in \mathbb{N}_0$, the function $D_q(e_m; x)$ is monotone decreasing with respect to $q \in (0,1)$.

This talk is based on the article [4], which is dedicated to the memory of Prof. G. M. Phillips.

Keywords: q-Bernstein operator, q-Durrmeyer operator, q-differences, shape-preserving property.

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Interval Valued Fuzzy Proximity Relations

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Abstract

Peters studied a proximal relator space $(X, \mathcal{R}_\delta)$ for a family of proximity spaces relations $\mathcal{R}_\delta$ on X [1]. Fuzzy sets are a useful mathematical structure that is defined by Zadeh [2]. There have been proposed several kinds of extensions such as intuitionistic fuzzy sets (IFS) [3], picture fuzzy sets (PFSs) Cuong and Kreinovich [4], interval-valued fuzzy sets (IVFS) [5] and so on. These fuzzy sets are used number of applications in many areas such as pattern recognition, image processing, medical diagnosis, decision-making and son on. Öztürk et al introduced fuzzy proximal relator spaces and defined fuzzy proximity relation to evaluate the proximity of the sets [6]. Afterward, they generalized fuzzy proximity to L-fuzzy proximity [7] and complex fuzzy proximity [8].

By using the concept of interval valued fuzzy relations, we forms interval-valued fuzzy relations on proximal relator spaces. In our paper, the interval-valued fuzzy proximity axioms that need to be satisfied by a interval-valued fuzzy relation on proximal relator spaces is given. Also, we defined spatial Lodato and descriptive Lodato proximity relations.

Keywords: Proximity space, fuzzy relation, fuzzy proximity, interval-valued fuzzy sets.

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$su(2)$ and $su(1,1)$ Operators within $N = 2$ Supersymmetry

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Abstract

A class of non-Hermitian Hamiltonian operators with real spectra that are expressed using $su(2)$ and $su(1,1)$ generators is studied [1]. The Lie algebra $su(2)$ is a fundamental concept in quantum mechanics, particularly in the context of angular momentum and spin. It is associated with the special unitary group $SU(2)$, which is the group of 2×2 unitary matrices with determinant 1. The Lie algebra $su(2)$ consists of all 2×2 traceless Hermitian matrices. In this study, $su(2)$ operators are studied in the context of supersymmetric quantum mechanics. Moreover, in quantum mechanics, the $su(1,1)$ Lie algebra plays a significant role, particularly in the context of systems with one-dimensional radial symmetry, such as the hydrogen atom or the Morse potential. This Lie algebra is relevant in these contexts because it describes the dynamical symmetries of these systems, particularly those associated with angular momentum. The connection between $su(1,1)$ and $su(2)$ algebras arises in certain contexts, particularly in the study of symmetries and dynamical properties of quantum systems. While the structures of the algebras are different, they often share similar mathematical properties and can sometimes be related through mathematical transformations or mappings. This connection has been explored in various areas of quantum mechanics and mathematical physics. This study is extended to $su(1,1)$ systems as well. As a toy model, the Swanson Hamiltonian [2] is studied. The construction of isospectral Hamiltonians is shown, and exact solutions for the corresponding systems are obtained.

Keywords: $su(2)$ Lie algebra, special functions, supersymmetric, graded algebra.

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Density of Quantized Approximations

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Abstract

The talk contains a review of non-trivial conditions on a set M in a Banach space X that are necessary or sufficient for the additive semigroup

$$R(M) = \{x_1 + \cdots + x_n \colon x_k \in M,\ n \in \mathbb{N}\}$$

to be dense in X. For example, if M is a rectifiable curve in a uniformly smooth real space X, and M does not lie entirely in any closed half-space, then $R(M)$ is dense in X. General results of this kind can be applied to approximation by simple partial fractions (logarithmic derivatives of polynomials) in various spaces of functions of a complex variable. For example, the well-known Korevaar's theorem can be derived from new general results on the density of a semigroup. We also present results on approximation by sums of shifts of one function, which are a natural generalization of simple partial fractions.

Keywords: Approximation, additive semigroup, density, Banach space, simple partial fractions, shifts, integer coefficients.

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Multiple Hopf Bifurcation of a Leslie Type Prey-Predator System with Allee Effect

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Abstract

In this presentation, we investigate the dynamic behavior of a continuous-time Leslie type prey-predator system [1] with the Allee effect. To characterize how the prey population grows in this model, we use an exponential growth function with the Allee effect. Initially, the local stability conditions of the positive equilibrium point of this system are determined. The prerequisites for the presence of Hopf bifurcation at this positive equilibrium point are next studied. This bifurcation is presented using Hopf bifurcation theory and normal form theory [2] using the Allee constant as a bifurcation parameter. We have observed that a gradually increasing Allee effect causes a supercritical Hopf bifurcation at the lower level of population density, while a strong Allee effect leads to a subcritical Hopf bifurcation. To put it another way, there is a stability switch from stable to unstable and from unstable to stable as the Allee effect increases [3]. Therefore, the system exhibits multiple Hopf bifurcations. Finally, we use numerical simulations to examine how the Allee effect affects the dynamics of both prey and predator populations. The findings that we have concluded have a number of important implications for future practice in understanding population behaviors and the natural survival strategies of endangered populations and in providing valuable insight into the interplay between population dynamics, ecology, conservation, and control theory.

Keywords: Stability analysis, Hopf bifurcation, periodic solutions, Allee effect.

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On Durrmeyer Variant of Mittag-Leffler Operators

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Abstract

The aim of this note is to introduce new positive linear operators involving the Mittag-Leffler function. Moreover, their moments and error estimation with the help of classical modulus of continuity have been established in this paper. Also, their A-statistical convergence property, (λ, γ)-statistical convergence property and rates of convergence in the Lipschitz spaces have also been discussed.

By the modified Mittag-Leffler function used by Ozarslan [1] we approximate certain class of functions and discuss an A-statistical approximation theorem for the following operators (1). For a sequence of real number (b_n), $\beta > 0$ is fixed. For $n \in \mathbb{N}$, the Mittag-Leffler operators are defined by

$$L_n^{(\beta)}(f,x) = \frac{1}{E_{1,\beta}\left(\frac{nx}{b_n}\right)} \sum_{k=0}^{\infty} f\left(\frac{kb_n}{n}\right) \frac{(nx)^k}{b_n^k \Gamma(k+\beta)} = \sum_{k=0}^{\infty} p_{n,k}(x,\beta) f\left(\frac{kb_n}{n}\right), \tag{1}$$

where $p_{n,k}(x,\beta) = \frac{1}{E_{1,\beta}\left(\frac{nx}{b_n}\right)} \frac{(nx)^k}{b_n^k \Gamma(k+\beta)}$ and $f \in E := \left\{ f \in C[0,\infty) : \lim_{n\to\infty} \frac{f(x)}{1+x^2} \text{ is finite} \right\}$. Here, as usual, $C[0,\infty)$ denotes the space of continuous functions defined on $[0,\infty)$. One should note that, the q-analogue of Mittag-Leffler operators given by (1) was introduced and studied by İcöz and Cekim [2]. Durrmeyer type generalization of Mittag-Leffler operators using Szász-Mirakjan basis function was investigated in [3]. Motivated by these works, we introduce and study the Durrmeyer variant of Mittag-Leffler operators in this note.

Keywords: Mittag-Leffler function, Durrmeyer operators, statistical approximation.

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Moyal-Higher-Spin Gauge Field Theory

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Abstract

Hard UV behaviour (small distances) in local relativistic quantum field theories is problematic not only because it leads to the need to perform infinite renormalisation, but more importantly because in some cases it stands in the way of proving mathematical existence of theories (a notable example is quantum electrodynamics). It is known that theories involving higher spin fields/particles can in principle solve the problem by softening the UV behavior, but an explicit implementation in terms of well-defined examples seems to be difficult (string theory achieves this, but at the high price of complicating the IR sector).

Building on our previous work [1, 2, 3] we show that in the Moyal Higher Spin gauge field theory based on the $2d$ non-commutative master space, an exponential softening of the UV behaviour is achieved without destroying the desired properties such as spacetime and gauge symmetries or the standard IR (large distances) behavior. The price to be paid is the sacrifice of strong locality, but this is expected in higher spin theories. The main problem of summation over a huge configuration space of states/fields with higher spin can be effectively solved by using Hermite functions and regularising sums over a complete set of functions as well as using a certain scaling in the perturbative approximation.

Keywords: Higher spin gauge theory, non-commutative gauge field theory.

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Interpolation on the Unit Circle for α-Harmonic Functions

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Abstract

A generalization of the deeply investigated harmonic functions, known as α-harmonic functions, have recently gained considerable attention. Similarly to the harmonic functions, an α-harmonic function u on the unit disc $\mathbb{D}$ is uniquely determined by its values on the boundary of the disc $\partial\mathbb{D}$. In fact, for any $z \in \mathbb{D}$, the value of $u(z)$ can be given as a contour integral over $\partial\mathbb{D}$ with a modified Poisson kernel. However, this integral can be difficult to evaluate, or the values on the boundary are known only empirically. In such cases, approximating $u(z)$ with an interpolatory formula, as a weighted sum of values of u at n nodes on $\partial\mathbb{D}$, can be an attractive alternative. The nodes and weights are to be chosen so that the degree d of exactness of the formula is maximized. In other words, the formula should be exact for all basis functions for α-harmonic functions of degree up to d, with d as large as possible.

In the case of harmonic functions, it is known that there is an interpolation formula of degree of exactness as large as $d = n - 1$. The objective of this paper are formulas of this type for α-harmonic functions. We will prove that, for certain α and given n, the degree of exactness cannot be $n - 1$, but there is a unique interpolation formula of degree $< n - 1$. Finally, we will prove convergence of such formulas to $u(z)$ as $n \to \infty$.

Keywords: Interpolation on the unit circle, modified Poisson kernel.

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Approximation Properties of Convolution Operators via Statistical Convergence Based on A Power Series

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Abstract

One of the main problems of analysis is to approximate a given function f by other functions which are good and simple. This problem has been studied by Weiertstrass and he has shown that every continuous real-valued function defined on $[a, b]$ can be written as a limit of polynomials. As it is well-known, the proof of this theorem is long and hard to follow and a simpler alternative proof has been given by Bernstein with the use of Bernstein operators. Noticing the basic properties of this operator; Bohman, Korovkin and Popoviciu have extended this result by a sequence of positive linear operators, independently. Then, the effect of positive linear operators is immediately noticed and these operators are well-studied in the approximation theory. One of the important class of such operators are convolution operators. Srivastava and Gupta have studied the approximation properties of summation-integral type operators in the classical sense. However, the classical limit does not need to exist and it is still possible to study approximation by statistical convergence. Positive convolution operators have been examined by Duman with the use of A-statistical convergence which is more general and stronger than the classical convergence.

In this talk, we present the approximation properties of convolution operators via P-statistical convergence which is not included by other methods given before. We also give the rate of this approximation. Furthermore, by providing examples such that the existing literature cannot be used, we support our results.

Keywords: Power series method, convolution operators, Korovkin type approximation.

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Direct and Inverse Theorems and the Rates of Convergence for the Generalized Positive Linear Operators

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Abstract

In this study, we will present some direct and inverse results on the finite interval for the sequence of generalized positive linear operator which was introduced in [3] and work out the rates of convergence properties of this operator.

Keywords: Sequence of positive linear operators, rates of convergence, modulus of continuity.

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A Seismic-Type Generalized Radon Transform

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Abstract

In this talk, we consider a generalized seismic Radon transform that maps a given function to its integrals over a certain family of curves in the plane similar to the one considered in [1]. Such transforms arise in many areas of mathematics, geophysics, and imaging science. This paper contains new explicit inversion formulas of this generalized seismic Radon transform for various families of curves based on their monotonicity. We derive an explicit inversion formula based on the use of the analogue of the Fourier slice theorem [2].

Keywords: Seismic Radon transform, reflection seismology, inversion formulas.

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Approximate Solutions of a Boundary Value Problem for Delay Nonlinear Difference Equations

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Abstract

The main object of investigation is a nonlinear difference equation with a delay. This type of equations, for example, find a wide application as models in the theory of automatic regulation, engineering, population dynamics, networks. A boundary value problem mixed with an initial condition for a nonlinear difference equation is studied. The main goal is to present an algorithm for constructing two sequences of successive approximations of the solution. This algorithm is based on the monotone iterative technique combined by the method of lower/upper solutions. Each term of the constructed sequences is a solution of an initial value problem for linear delay difference equations which solutions could be easily obtained. Also the terms of the sequences are lower/upper solutions of the given problem. It is proved both sequences are monotonically convergent. The suggested procedure is simulated and computer realized on some examples.

Keywords: Difference equation, delay, boundary value problem, approximate solutions.

Acknowledgments: The first author is partially supported by the fund of FNI Plovdiv University "P. Hilendarski" under project MUPD23-FMI-009.

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Study of the Approximate Solution of the Delay Fourth Kind Integral Equations

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Abstract

Fourth kind integral equations or Integral-Algebraic Equations (IAEs) are the coupled systems of the first and second kind Volterra integral equations. The solutions of these equations with non-vanishing delays can be include primary discontinuity points such that at these points the solution of the IAEs will exhibit a low degree of regularity [1, 2, 3]. In this paper, we study the piecewise approximate collocation method for the delay fourth kind integral equations based on dividing the definition domain into several subintervals according to the primary discontinuous points associated with the delay function. Some well-established results in terms of regularity, existence, uniqueness, and also convergence of the solution for the problem under study have been presented masterly. Finally, some test problems have been fairly well-studied, for the sake of verifying theoretical achievements in practice.

Keywords: Fourth kind integral equations, delay integral-algebraic equations, primary discontinuity points, piecewise collocation methods, error analysis.

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New Generalization of λ-Bernstein Kantorovich Operators for Better Error Estimation

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Abstract

In this paper, we introduce a new family of generalized λ-Bernstein Kantorovich operators and investigate their approximation properties. Furthermore, we compare the operators introduced in this paper with the λ-Bernstein Kantorovich operators and demonstrate that the newly introduced operators yield better results. Additionally, we study the uniform convergence and rate of convergence of these operators in terms of the first and second order modulus of continuity. Finally, we provide numerical examples and graphs that support the results obtained in this study.

Keywords: Bernstein operators, Bernstein-Kantorovich operators, polynomial approximation, λ-Bernstein Kantorovich operators, rate of convergence, modulus of continuity, uniform convergence.

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On the Linearization Coefficients of the Generalized Bessel Polynomials

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Abstract

In this paper, we compute linearization coefficients of the $x^m p_n(x)$ and $p_n(x)q_m(x)$ expansions in terms of the generalized Bessel polynomials, where the $p_n(x)$ and $q_m(x)$ are the n and m degree polynomials, respectively. We also take certain types of orthogonal polynomials such as Laguerre and Legendre polynomials instead of $p_n(x)$ and $q_m(x)$ and discuss their special cases.

Keywords: Linearization coefficients, orthogonal polynomials, hypergeometric function.

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A Generalization of Jakimovski-Leviatan Operators

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Abstract

In this talk, we provide a generalization of the Jakimovski-Leviatan operators using hypergeometric functions. We look into the convergence rate and approximation properties. We illustrate theoretical findings by using numerical examples. Finally, we offer a graphic comparison between the recently constructed operators and the Jakimovski-Leviatan operators.

Keywords: Jakimovski-Leviatan operators, hypergeometric function.

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Orthogonalizing q-Bernoulli Polynomials

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Abstract

In this study, we utilize the Gram-Schmidt orthogonalization method to construct a new set of orthogonal polynomials called $OB_n(x,q)$ from the q-Bernoulli polynomials. We demonstrate the relationship between $OB_n(x,q)$ polynomials and the little q-Legendre polynomials, and derive a generalized formula for $OB_n(x,q)$ by leveraging the little $q-$Legendre polynomials.

Keywords: q-Bernoulli polynomials, q-Legendre polynomials, orthonormal polynomials.

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Approximation Theory and Special Functions
ATSF 2024 Conference - 8th Series
September 4-7, 2024, Ankara - Türkiye

New Cubature Formulae for Sobolev Spaces W_q^α with a Dominant Mixed Derivative

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Abstract

We present new cubature formulae for multivariate periodic functions f w.r.t each variable with period 1 and with some smoothness $\alpha > 0$

$$F_m(f;p) = \frac{1}{p^m} \sum_{\substack{|\mathbf{k}|_1 = m \\ \mathbf{k} \in \mathbb{Z}_+^n}} \sum_{r_1=0}^{p^{k_1}-1} \cdots \sum_{r_n=0}^{p^{k_n}-1} \prod_{j=1}^{n-1} \frac{\sum_{l=1}^{p-1} e^{-\pi i \left(\frac{2lr_j}{p} + \varepsilon(k_j) \right)}}{(p-1)^{1-\varepsilon(k_j)}} f\left(\frac{r_1}{p^{k_1}}, \cdots, \frac{r_n}{p^{k_n}} \right), \tag{1}$$

where $p > 1$ is prime, $m \in \mathbb{N}$, $\varepsilon(x) = 1$ if $x \geq 1$ and $\varepsilon(x) = 0$ else.

The new cubature formulae are exact for polynomials with frequencies from the corresponding hyperbolic cross, and the upper bound of the worst-case error is $O(N^{-\alpha} (\log N)^{(\alpha+1/\tilde{q})(n-1)})$ for periodic functions from Sobolev spaces W_q^α, where N-number of nodes, $1 \leq q \leq \infty$, $\tilde{q} = \min\{q, 2\}$, n-dimension.

Keywords: Cubature formulae, multivariate numerical integration, Sobolev space, periodic functions, a dominant mixed derivative.

Approximation on an Infinite Interval in Power Series Statistical Sense

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Abstract

In this presentation, we study the problem of approximation on an infinite interval via statistical convergence with respect to power series methods. We first introduce the Korovkin type approximation theorem in this setting. Then, an example is given such that our new approximation result provides but its classical and statistical cases do not work. Finally, we present the rate of convergence.

Keywords: Power series method, statistical convergence, Korovkin theorem.

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Properties of (bo)-Fragments on Lattice Normed Vector Lattices

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Abstract

The concept of fragments was first introduced by Dutch mathematician H. Freudenthal. This concept has been used to investigate orthogonally additive operators within vector lattices. Fragments have been crucial in examining the characteristics of these operators. M. Pliev expanded this concept to lattice normed spaces, where these fragments are called (bo)-fragments.

In this talk, we will first explore the relationship between fragments and (bo)-fragments. Additionally, we will present various properties of (bo)-fragments. We will demonstrate that, under specific conditions, the collection of all (bo)-fragments of a given positive element forms a Boolean algebra. Furthermore, we will establish that this collection is order complete. Finally, We will discuss the properties of fragments in the space $C[0,1]$, highlighting their significance in understanding the (bo)-fragments of $C[0,1]$. This presentation is based on the findings and discussions presented in [1].

Keywords: Lattice normed vector lattices, fragment, (bo)-fragment.

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On Better Approximation Order for the Nonlinear Meyer-König and Zeller Operator of Maximum Product Kind

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Abstract

Bede et al. introduced nonlinear max-product operators by using the maximum instead of the sum, and they examined some approximation properties of these operators (see [2]). The nonlinear Meyer-König and Zeller operator of maximum product kind was one of these operators. In [1], they examined the approximation and shape preserving properties of this operator. Additionally, they found the order of approximation to be $\frac{(1-x)\sqrt{x}}{\sqrt{n}}$ by the modulus of continuity and claimed that this order of approximation could not be improved except for some subclasses of the functions. Contrary to this claim, we show that a better order of approximation can be obtained without reducing the function class by the classical modulus of continuity.

We find the order of approximation to be $\frac{(1-x)x^{1/\alpha}}{n^{1-1/\alpha}}$, $\alpha = 2, 3, \ldots$. Since $1 - \frac{1}{\alpha}$ tends to 1 for enough big α, therefore, we improve this degree of approximation.

Keywords: Nonlinear Meyer-König and Zeller operator of maximum product kind, modulus of continuity.

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Umbral Calculus, Hypergeometric Functions and Applications

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Abstract

Umbral calculus and Operator Theory, through the use of the integral transforms, offer a flexible mechanism in the handling of special functions, orthogonal polynomials, families of hypergeometric functions, PDE solution, in fractional dynamic too and in their applications. From biology to particle physics, from deep learning to analysis of electrical circuits from a quantum point of view, indeed, operatorial methods find countless areas of development. In this context, the flexible use of the mathematical tools mentioned above will provide various solutions explored through numerous examples in pure and applied mathematics.

Keywords: Umbral methods, operators theory, special functions, Bessel function, hypergeometric functions, trigonometric functions, error function, Gaussian function, integral calculus, deep learning.

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Ulam Type Stability for Caputo Type Fractional Delay Differential Equations

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Abstract

In this paper will consider differential equations with generalized fractional derivatives of Caputo type. We will discuss the application of Ulam type stability to initial value problems (IVP) as well as to boundary value problems (BVP) for the studied fractional differential equation. We emphasize on some misunderstandings when Ulam type stability is applied to BVP. We suggest one possible way to avoid these misunderstandings by including a parameter in the boundary condition. The existence of the solution depending on this parameter is proved. Later, in the application of Ulam type stability, the parameter is chosen in appropriate way such that the solution of the studied fractional equation to depend significantly on the arbitrary chosen solution of the corresponding inequality. Some examples will illustrate the theoretical results.

Keywords: Caputo type fractional derivative, fractional differential equation, delay, initial value problem, boundary value problem, Ulam type stability.

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The (M, λ_n^π) Method of Summability and Determination of the Class of Saturation

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Abstract

In this study, our aim is to define the (M, λ_n^π)-method of summability for a sequence of positive numbers $\pi = \{\pi_n\}$ and a sequence of real numbers $\lambda = \{\lambda_n\}$. Also, we shall examine necessary and sufficient conditions for this method. On the other hand, we obtain various saturation classes with the L_p-norm for this method and give some characterizations.

Keywords: (M, λ_n^π) method, Abel-summability, Fourier Series, saturation class.

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Scattering Properties of the Impulsive Dirac Systems with a Particle Mass

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Abstract

In this talk, we will investigate the Jost solution and unbounded solution of the impulsive Dirac systems with a particle mass m. Also, we will study analytic and asymptotic properties of these solutions. Furthermore, characteristic properties of the scattering function of the impulsive Dirac systems will be examined. Finally, we will present the asymptotic behavior of the impulsive unperturbed Dirac systems by using obtained Jost function and scattering function of the system.

Keywords: Differential equations, Dirac systems, Jost solution, scattering function.

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Modified Jakimovski-Leviatan Type Szász Operators

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Abstract

We construct modified Jakimovski-Leviatan type Szász operators $S_n^{\alpha,\beta,\gamma}(f;x)$ with the help of Gamma transform and generalized Jakimovski-Leviatan type Szász operators which contain Sheffer polynomials. First, we calculate the moments in terms of the Stirling numbers of second kind and then, we give the rate of convergence with the help of first modulus of continuity. Also, we obtain a global rate of convergence by using certain Lipschitz type function space. Using the method in [4], we give better estimation for the operators $S_n^{\alpha,\beta,\gamma}(f;x)$. Furthermore, graphical representations and numerical examples are presented that support the convergence of $S_n^{\alpha,\beta,\gamma}$ to $f(x)$.

Keywords: Jakimovski-Leviatan operators, Szász operators, Sheffer polynomials, modulus of continuity, Lipschitz type function space, Stirling numbers of the second kind.

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Fixed Point Results of Contractive Mappings Under Simulation Function in b-Metric Spaces

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Abstract

In this paper, we give a new fixed point result in b-metric space via simulation function which extends some important results in the literature.

Keywords: Simulation function, b-mertic space, contractive mapping.

References:

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On the Zero Location of Appell Sequences and Their Cognate Sequences

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Abstract

Given a Sheffer sequence of polynomials, we introduce the notion of an associated sequence called the cognate sequence. We study the relationship between the zeros of this pair of associated sequences and show that in case of an Appell sequence, as well as a more general family of Sheffer sequences, the zeros of the members of each sequence (for large n) are either real, or lie on a line $\Re z = c$. In addition to finding the zero locus, we also find the limiting probability distribution function of the zeros of such sequences. Finally, we characterize all real entire functions $g(z)$ such that the zeros of the Appell sequence $\{p_n(s)\}$ generated by the function $g(z)e^{sz}$ lie on a vertical line. This work is a continuation of our previous investigations of Sheffer sequences and their zeros conducted in [1].

Keywords: Appell sequence, Sheffer sequence, cognate sequence, zero locus, limiting distribution.

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Discrete and Matrix Analogues of Hypergeometric Series

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Abstract

Recent research on discrete hypergeometric series and matrix-parameter hypergeometric series will be presented, including some open questions and directions for future research. In particular, difference equations and applications for the discrete functions and norm lower bound estimates and divergence criteria for the matrix-valued functions will be discussed.

Keywords: Discrete hypergeometric, matrix hypergeometric, difference equation, lower bound estimates

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Approximation Theory and Special Functions
ATSF 2024 Conference - 8th Series
September 4-7, 2024, Ankara - Türkiye

Suborbital Graphs of the Normalizer of $\Gamma_1(n)$ in $\mathrm{PSL}(2,\mathbb{R})$

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Abstract

This study uses a bridge between analysis and algebra. Let $\mathrm{PSL}(2,\mathbb{R})$ denote the group of all linear fractional transformations (are also called Möbius transformations)

$$T : z \to \frac{az+b}{cz+d},$$

where a, b, c, and d are real and $ad - bc = 1$. This is the automorphism group of the upper half plane $\mathbb{H} := \{z \in \mathbb{C} : \mathrm{Im}(z) > 0\}$. The modular group Γ is the subgroup of $\mathrm{PSL}(2,\mathbb{R})$ such that a, b, c, and d are integers. $\Gamma_1(n) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \equiv \pm \begin{pmatrix} 1 & * \\ 0 & 1 \end{pmatrix} \bmod n \right\}$ is a subgroup of Γ. The normalizer of $\Gamma_1(n)$ in $\mathrm{PSL}(2,\mathbb{R})$ is

$$M = \begin{cases} \begin{pmatrix} Qx & y \\ nz & Qw \end{pmatrix}_{\det=Q}, & \text{if } n \neq 4 \\ \begin{pmatrix} e & f/2 \\ 2g & h \end{pmatrix}_{\det=1}, & \text{if } n = 4 \end{cases}$$

where Q is a Hall divisor of n (i.e. $(Q, n/Q) = 1$) and x, y, a, w, e, f, g, h are all integers. All these groups are basics of the theory of elliptic modular functions. It is known that studying the idea of a group G acting on a set Ω, we can also establish some additional structure on Ω. One of these structures is a graph. The connection between transitive groups and graphs give us new insight into some known results such as in [1, 2]. Here we also used this connection. The suborbital graph is a graph arisen from the transitive group action. It was shown that suborbital graph is an useful tool in the study of the modular group, then some other finitely generated groups have been studied by suborbital graphs. Since the group structure of the normalizer is much more complex than the modular group, its suborbital graphs have been studied under various restrictions. In [3], it is given the group structure of normalizer of some specific congruence subgroups in $\mathrm{PSL}(2,\mathbb{R})$. In this study we investigate to suborbital graphs arisen from the normalizer of $\Gamma_1(4)$ in $\mathrm{PSL}(2,\mathbb{R})$.

Keywords: Suborbital graphs, normalizer, congruence subgroups.

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A New Study on a Random Conformable Fractional System Modeling Acid-Mediated Tumor Cell Invasion

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Abstract

The ability of tumor cells to spread is directly influenced by the pH level of the tumor microen- vironment. An acidic pH gradient is predicted by a mathematical model developed by Gatenby et al. to extend into the peritumoral normal tissue. Normal mammalian cells are acidic intersti- tial pH intolerant within the range usually seen within this gradient, while tumor cells are not. In this talk, the model proposed by Gawlinski et al and developed by M¨arkl et al including the impact of crowding effects is considered as a fractional system. Moreover, the model is developed to contain a random component with Gamma distribution for the tumor cell invasion and an anal- ysis is conducted on the conformable fractional system contaning the numerical characteristics of tumor cell invasion. In order to obtain the approximate solutions for the model the conformable fractional q-Shehu homotopy analysis transform method is utilized. In terms of qualitative facts obtained through experimentation, the numerical solution is seen to validate the model predictions. Furthermore, the system is analyzed numerically in terms of arbitrary derivative orders.

Keywords: Conformable fractional derivative, Shehu transform, tumor model, gamma distrubition.

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On Some Comparision of Multistep Multiderivative Methods and Its Application to Solve the Volterra Integro-Differential Equations

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Abstract

The need to solve integro-differential equations of Volterra type arises in almost all fields of natural sciences. For this purpose, the use of Multistep Multidifferential Methods with constant coefficients is proposed, some concert methods are established and it is shown how they can be applied to solve the initial-value problem for Volterra integro-differential equations. Furthermore, it is shown how the relation between the solution of the initial-value problem for both ordinary differential and Volterra integro-differential equations can be used. For the construction of more exact numerical methods, it is proposed here to use some partial cases of multistep multidifferential methods. The concepts of stability and degree of numerical methods are used to compare these methods and optimal methods are constructed by using the maximum value of the degree for stable methods. The construction of some others is suggested to use the stability region. As is known, there are some stable methods whose stability region is equal to zero, i.e. this region is consist of one point. Usually predictor-corrector type methods are used to expand the stability region. Here, it is shown how optimal methods can be applied to solve initial-value problems for Volterra integro-differential equations. We consider the case where the kern of integral is the degree rate function, and establish special numerical methods for their solutions, and also show their advantages.

Keywords: Multistep multiderivative methods, stability and degree, the Volterra integro-differential equation, initial-value problem, optimal method.

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Investigation of the Biological Characteristics of Tuberculosis and Dengue Disease Model Using a Novel Hermite Wavelets Based Numerical Approach

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Abstract

This study explores the implementation of a wavelet-based technique to resolve two mathematical models. The initial model addresses the propagation of Dengue fever, taking into account various factors like transmission rate and mortality rate. The subsequent model emphasizes the spread of Tuberculosis, including different control parameters. To address these systems of nonlinear differential equations, we use the Hermite wavelet collocation method (HWCM), which leverages the operational matrix of integration of Hermite wavelets. This method translates the initial nonlinear differential equations into solvable algebraic equations. Thereafter, the numerical solution is derived using the Newton-Raphson method. We compare the outcomes of the proposed method with analytical solutions, solutions obtained using Bernoulli wavelets, and the Runge-Kutta method. In addition, we explore several theorems to assess the convergence of the proposed method. Our findings show that the proposed approach attains higher accuracy and lower error rates than current methods.

Keywords: Tuberculosis model, Dengue disease model, Hermite wavelet, collocation technique, MATLAB.

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A New Generalization of Sigmoid Polynomials and Its Properties

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Abstract

In this talk, we define a new generalization of sigmoid polynomials and numbers. Starting from this definition, we give some properties of these polynomials and numbers. Moreover, we give some relations of these polynomials with some existing polynomials in the literature.

Keywords: Sigmoid polynomials, sigmoid numbers, generating function.

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Hardy Inequalities for Fractional (k,a)-Generalized Harmonic Oscillator

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Abstract

We will define a-deformed Laguerre operators $L_{a,\alpha}$ and a-deformed Laguerre holomorphic semigroups on $L^2\left((0,\infty),d\mu_{a,\alpha}\right)$. Then we give a spherical harmonic expansion, which reduces to the Bochner-type identity when taking the boundary value $z=\frac{\pi i}{2}$, of the (k,a)-generalized Laguerre semigroup introduced by S. Ben Saïd, T. Kobayashi and B. Ørsted [1]. And then we prove a Hardy inequality for fractional powers of the a-deformed Dunkl harmonic operator $\triangle_{k,a}:=|x|^{2-a}\triangle_k-|x|^a$ using this expansion. When $a=2$, the fractional Hardy inequality reduces to that of Dunkl-Hermite operators given by Ó. Ciaurri, L. Roncal and S. Thangavelu [2]. The operators $L_{a,\alpha}$ give a tangible characterization of the radial part of the (k,a)-generalized Laguerre semigroup on each k-spherical component $\mathcal{H}_k^m\left(\mathbb{R}^N\right)$ for $\lambda_{k,a,m}:=\frac{2m+2\langle k\rangle+N-2}{a}\geq -1/2$ defined via decomposition of unitary representation.

Keywords: Laguerre holomorphic semigroup, spherical harmonic expansion, decomposition of unitary representation, Bochner-type identity, fractional Hardy inequality.

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Generalized k-Cesàro Polynomials

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Abstract

The main purpose of this study is to introduce k-analogue of the generalized Cesàro polynomials and give their some properties. Firstly, generalized k-Cesàro polynomials are defined with the help of a generating function relation. After getting the explicit form of these polynomials, various generating function relations, the Mellin transform formula, expressions in terms of k-hypergeometric function and some integral representations are obtained.

Keywords: Generalized Cesàro polynomials, k-analysis, generating function, fractional derivative operator, Mellin transform formula, integral representation.

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Fuzzy Parameterized CR-Fuzzy Soft Sets

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Abstract

Numerous applications of classical mathematics deal with uncertain data. Several methods have been employed to manage uncertainties. Zadeh's [6] discovery of fuzzy set theory in 1965 is the most significant of these tools. A lot of research on the generalizations of fuzzy sets were carried out following the development of fuzzy sets theory. One of these is the Molodtsov [4] discovered soft set theory. After Molodtsov's discovery Maji et al. [3] defined fuzzy soft sets. After that, Cagman et al. defined fuzzy parameterized fuzzy soft set and also they redifened the fuzzy soft set and their operations. See for this [1] and [2] respectively. Eventually, in 2023 Salih and Ibrahim [5] developed CR-fuzzy sets, a new class of generalized fuzzy sets.

In this study, in light of these findings, we shall construct a new tool. We will give definitions for fuzzy parameterized CR-fuzzy soft sets as well as some actions between them.

Keywords: Fuzzy sets, soft sets, CR-fuzzy soft sets, fuzzy parameterized fuzzy soft sets.

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Spectrum Density Estimation of Sample Covariance Matrices with Correlated Entries

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Abstract

We focus on kernel estimator of the density function of the limiting spectral distribution of sample covariance matrix associated firstly, to a large class of weak dependent sequences of real-valued random variables having only moment of order 2. Afterwards, sample covariance matrices of the block-independent model and the random tensor model, where the data does not have independent coordinates. In the last two cases, we show that the kernel estimator of the density function converges to the Marchenko-Pastur density with probability one.

A simulation study is conducted to show the performance of the kernel estimators of the density function and then compare these estimators with the one obtained by the Stieltjes transform method.

Keywords: Weak dependence, Stieltjes transform, Marchenko-Pastur distribution.

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α-Analogue of Bessel Function

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Abstract

In this talk, we propose α-analogue of Bessel equation whose coefficient functions are determined in terms of α-power functions [1, 2]. We construct its solution, namely α-Bessel function by Frobenius method. Additionally, we present the properties of α-Bessel function and its reductions to well known continuous and discrete Bessel functions .

Keywords: α-Time scale, power function, Bessel equation, Bessel function.

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Concave Generalizations of Delta and Nabla Bennett-Leindler Type Dynamic Inequalities

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Abstract

Delta and nabla Bennett-Leindler type dynamic inequalities are extended via concavity. The unification characteristic of delta and nabla inequalities merges the results obtained for their special cases in continuous and discrete settings, where these particular results have appeared in the literature for the first time as well. If concavity condition is removed, these inequalities provide generalizations of delta and nabla Bennett-Leindler type dynamic inequalities [1].

Keywords: Nabla time scale calculus, Hardy-Copson inequality, Bennett-Leindler inequality, concavity.

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General Families of Cosine and Sine Appell Polynomials

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Abstract

In this study, we introduce cosine and sine Appell polynomials and investigate their quasi-monomiality properties, recurrence relations, lowering and raising operators, differential equations, determinantal representations and summation formulas. This new definition includes some potentially usefull subfamilies of polynomials such as cosine and sine unified Apostol-type polynomials, cosine and sine Gould-Hopper, Laguerre and truncated exponential based unified Apostol-type polynomials. The particular cases of the main results are also given for these subfamilies.

Keywords: Appell polynomials, general Appell polynomials, Apostol-type polynomials, Gould-Hopper polynomials, Laguerre polynomials, truncated exponential polynomials.

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(Poster Presentation)

Spectral Method for a Particular Case of the Heat Convection-Diffusion Equation

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Abstract

The aim of this paper is to investigate the Legendre spectral method for solving a particular case of the heat convection-diffusion equation, formulated by the mixed initial-boundary value problem in a finite regular set $\Lambda = (-1, 1)$, we use some techniques to convert the problem to a system of ordinary differential equations and by an analysis matricial we find a general term defines all ordinary differential equations of this system, we solve this general term we get the desired approximate solution, we also present the error estimate.

The concerned problem refers to the equation:

$$\begin{cases} \partial_t u(x,t) - \partial_x^2 u(x,t) + \partial_x u(x,t) + u(x,t) = f(x,t), & x \in \Lambda,\ t > 0 \\ u(-1,t) = u(1,t) = 0 & t > 0 \\ u(x,0) = u_0(x) & x \in \Lambda \end{cases}$$

(see [1, 2, 3]). For more details and complete analysis of the spectral methods we refer to [4, 5, 6].

Keywords: Heat convection-diffusion equation, spectral method, orthogonal polynomials, error sstimate.

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(Poster Presentation)

(p, q)-Compactness in Spaces of Holomorphic Mappings

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Abstract

Based on the concept of (p, q)-compact operator for $p \in [1, \infty]$ and $q \in [1, p^*]$, we introduce and study the notion of (p, q)-compact holomorphic mapping between Banach spaces. We prove that the space formed by such mappings is a surjective $pq/(p+q)$-Banach bounded-holomorphic ideal that can be generated by composition with the ideal of (p, q)-compact operators. In addition, we study the Mujica's linearization of such mappings, its relation with the $(u^*v^* + tv^* + tu^*)/tu^*v^*$-Banach bounded-holomorphic composition ideal of the (t, u, v)-nuclear holomorphic mappings for $t, u, v \in [1, \infty]$, its holomorphic transposition via the injective hull of the ideal of $(p, q^*, 1)$-nuclear operators, the Möbius invariance of (p, q)-compact holomorphic mappings on $\mathbb{D}$, and its full compact factorization through a compact holomorphic mapping, a (p, q)-compact operator and a compact operator.

Keywords: Vector-valued holomorphic function, linearization, factorization theorems, (p, q)-compact operator, (p, q)-compact holomorphic mapping, (t, u, v)-nuclear holomorphic mapping.

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(Poster Presentation)

On Quotients of Ideals of Bounded Holomorphic Maps

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Abstract

Let $\mathcal{I}, \mathcal{J}$ be operator ideals and let E, F be Banach spaces. Following [3, p. 132], a bounded linear operator $T : E \to F$ is said to belong to the left-hand quotient $\mathcal{I}^{-1} \circ \mathcal{J}$, and we write $T \in \mathcal{I}^{-1} \circ \mathcal{J}(E, F)$, if $S \circ T \in \mathcal{J}(E, G)$ for all $S \in \mathcal{I}(F, G)$, where G is an arbitrary Banach space. The right-hand quotient $\mathcal{I} \circ \mathcal{J}^{-1}$ is defined in a similar way. It is well known that $\mathcal{I}^{-1} \circ \mathcal{J}$ and $\mathcal{I} \circ \mathcal{J}^{-1}$ are operator ideals (see [2, 3.2.2]). Furthermore, if $[\mathcal{I}, \|\cdot\|_{\mathcal{I}}]$ and $[\mathcal{J}, \|\cdot\|_{\mathcal{J}}]$ are Banach operator ideals, and we set

$$\|T\|_{\mathcal{I}^{-1}\circ\mathcal{J}} = \sup\{\|S \circ T\|_{\mathcal{J}} : S \in \mathcal{I}(F, G), \|S\|_{\mathcal{I}} \leq 1\},$$

for every $T \in \mathcal{I}^{-1} \circ \mathcal{J}(E, F)$, then $[\mathcal{I}^{-1} \circ \mathcal{J}, \|\cdot\|_{\mathcal{I}^{-1}\circ\mathcal{J}}]$ is a Banach operator ideal by [2, 7.2.2].

Based on this notion, we introduce and study the concept of bounded-holomorphic left-hand quotient $\mathcal{I}^{-1} \circ \mathcal{J}^{\mathcal{H}^\infty}$, where $\mathcal{I}$ is an operator ideal and $\mathcal{J}^{\mathcal{H}^\infty}$ is a bounded-holomorphic ideal. We show that such quotients are a method for generating new bounded-holomorphic ideals. In fact, if $\mathcal{J}^{\mathcal{H}^\infty}$ has the linearization property in an operator ideal $\mathcal{A}$, then $\mathcal{I}^{-1} \circ \mathcal{J}^{\mathcal{H}^\infty}$ is a composition ideal of the form $(\mathcal{I}^{-1} \circ \mathcal{A}) \circ \mathcal{H}^\infty$ presented in [1]. We also introduce the notion of Grothendieck holomorphic map and prove that they form a bounded-holomorphic ideal which can be seen as a bounded-holomorphic left-hand quotient. In the same way, the ideal of holomorphic maps with Rosenthal range can be generated as a bounded-holomorphic left-hand quotient.

Keywords: Vector-valued holomorphic mapping, linearization property, bounded-holomorphic left-hand quotient, bounded-holomorphic ideal.

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(Poster Presentation)

The Definition and Properties of Levinson's Functional on Time Scale

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Abstract

The construction of Levinson's functional on time scale, as a difference between the right-hand side and left-hand side of the Levinson inequality on time scale, brings the opportunity to investigate known time scale integral inequalities in other directions using properties and boundaries of new functionals.

The Levinson functional on time scale and its properties of superadditivity and monotonicity will be investigated. Obtained properties will be used to derive the bounds of the given Levinson's functional which provides a refinement and the converse of known Levinson's inequality on time scale. Further, new types of functionals, using weighted generalized and power means on time scale, will be defined and their properties, which can be employed in future works to obtain refinements and converses of known integral inequalities on time scale, will be proved.

Keywords: Levinson's inequality, Jensen's functional, time scale calculus.

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(Poster Presentation)

Certain Fractional Integral Inequalities and the Mittag-Leffler Function

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Abstract

Many researchers have been motivated by applications of inequalities in fractional calculus to investigate extensions and generalizations using various fractional differential and integral operators. The Mittag–Leffler function is of particular importance, which with its generalizations appears as a solution of differential or integral equations of fractional order.

Several types of inequalities in the fractional calculus for the class of $(h, g; m)$-convex functions containing the extended generalized Mittag–Leffler function in the kernel of applied fractional integral operators will be presented.

Keywords: Fractional calculus, Mittag-Leffler function, inequality.

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(Poster Presentation)

Chlodowsky Variant of Bernstein-Type Operators on the Unit Disk

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Abstract

In this paper, we define the shifted nth Bernstein-Chlodowsky operators. We studied approximation properties of these operators.

Keywords: Bernstein-Chlodowsky operator, approximation, shifted nth Bernstein–Chlodowsky operator.

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(Poster Presentation)

A New Type of Soft Multi Rough Sets

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Abstract

Soft multi rough sets which are a hybrid model combining rough sets with soft multisets are defined by using soft multi rough approximation operators. Soft multi rough sets can be seen as a generalized rough set model based on soft multisets. In this study, we present a new type of soft multi rough set and its basic properties. We define a new type of soft multi upper approximation operator by means of soft multi neighborhoods. We show that our soft multi upper approximation operator is smaller than with other type of soft multi upper approximation operator. We found that it has more properties than the existing one.

Keywords: Rough set, soft multiset, soft multi rough set.

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